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The Information Basis of Multivariate Poverty Assessments[1]

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1.1 Introduction

Evaluation of household or individual well-being is now widely accepted as a multi-attribute exercise. Far less agreement exists on such matters as which attributes to include, how such attributes are related and/or contribute to overall well-being, and what criteria to employ for complete (that is, index-based) ranking of well-being situations. Some degree of robustness may be sought through weak uniform rankings of states, as by stochastic dominance and related criteria. A useful starting point, both for the believers and non-believers in the multidimensional approach, is to see the traditional univariate assessments in the multiattribute setting: it is as though a weight of one is attached to a single attribute, typically income or consumption, and zero weights given to all other real and potential factors! Univariate approaches do not avoid, they rather impose very strong a priori values.

Given a matrix X of attributes, with typical element x_{ij}, for units $i = 1, 2, \dots, n$ and attributes $j = 1, 2, \dots, m$ any scalar measure of well-being $f(x)$ is a function $f(.):\mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$. It is evident, and inescapable, that $f(.)$ aggregates over both individuals and attributes. In so doing, it must assign weights to both individuals and to each attribute. In addition, every $f(.)$ implies a certain relation between individuals as well as attributes. There are only two choices before us: make these functional characteristics explicit, or allow to implicitly derive them from other considerations. Viewed this way, an axiomatic characterization of 'ideal' poverty (and other) measures does well to explicate the properties of $f(.)$ with respect to individual weights and relations, but not the aggregation over attributes. Similarly, an axiomatic characterization of ideal aggregation measures may produce welfare-theoretic features that may not be desired. There is no minimalist set of axioms commanding universal acceptance which may produce even a family of functions $f(.)$. Additional, more restrictive and less acceptable properties must be imposed to justify any one measure $f(.)$.

A deeper understanding of indices, be they of poverty or inequality, makes clear that all indices are functions of the distribution of the desired attribute(s). Put another way, any index is a function of the moments of the distribution of the attributes. As such, all indices omit more or less information relative to the full distribution.

Only one function, the characteristic (or moment-generating) function, is equivalent to the whole distribution. Entropy comes close, see Ebrahimi, Maasoumi and Soofi (1999a,b), since two entropies are equal if, and only if, the two underlying distributions are the same. This property of entropy and other information measures of welfare seems to be poorly appreciated by economists. For instance, there exists no better or more complete measure of 'divergence' between a given income distribution and the uniform (rectangular) distribution. Put another way, there cannot exist a more complete and more fully informed measure of equality/inequality than entropy. Only if we additionally restrict such indices can we justify other measures.[2] Many of these additional restrictions and properties are sensible. But they are almost never consensus properties. This comment generally applies to the whole edifice of welfare function-welfare-theoretic assessments and the restrictions that derive from it, such as 'individualistic', 'utilitarian', and 'welfarist' social welfare function (SWF) basis for the discussion of indices. While the latter provides the most disciplined and elegant formalism for analysis, it does not have a claim to producing the most complete and most 'informed' indices, as we shall see.

The literature on multidimensional poverty recognizes three broad classes of measures (Deutsch and Silber, 2005): The fuzzy set approach, the information theory approach, and the axiomatic approach to poverty measures. As argued above, all three must produce aggregate measures of well-being, or what we may term 'individual representation functions'. In the end, poverty measures derive from this aggregate and the distribution of the constituent attributes. All measures classify certain members of the population as poor, and may assess the intensity of their poverty (such as the expected shortfall). In this chapter we adopt the information theory perspective to assess the different aggregation methods, explicit or implicit, and examine who is classified as poor in the axiomatic and the information theory approaches.

A brief description of the Information Theory (IT) approach is as follows: employing information functions and related entropies, divergence/distance between distributions is a well defined concept in IT. Following Maasoumi (1986), we find individual-level aggregate welfare functions whose distributions are the least divergent from the distributions of the constituent welfare attributes. This provides a method of optimal aggregation in the multidimensional welfare context that is able to subsume all existing implicit aggregators in this field, but also suggest new ones. The second step is then to measure 'poverty' in the distribution of this aggregate function of well-being. All of the existing univariate poverty measures present as candidates. The IT approach also opens new vistas in terms of the definition and concept of *the poverty line* in the multidimensional context. Several definitions and approaches emerge which go beyond the existing methods.

We conclude with an empirical example and some remarks concerning implementation and practical issues. One issue concerns the identification of truly distinct dimensions or attributes. This highlights, again, the statistical role played by any chosen index and its ability to utilize information in different dimensions. This is both instructive, and illuminating in terms of the 'information completeness' of an index alluded to above, but is not entirely unique to the multidimensional context, merely aggravated by it. Since we only consider three dimensions – income, education and health – in our application to Indonesian data, in this chapter we do not deal with the clustering techniques that also use consistent IT method for dimension reduction based on the similarity of the attribute distributions. We merely report several robust measures of dependence between our chosen attributes to shed light on their relations.

1.2 Multivariate poverty measures

Poverty analysis is concerned with the lower part of the distribution of well-being. In particular, the measurement of poverty generally involves three steps: first, selecting an appropriate indicator to represent individuals' well-being; then, choosing a poverty line which identifies the 'lower part' of the distribution to be the object of study, and hence to categorize people as poor and non-poor; and, finally, selecting a functional to aggregate individuals.

The monetary approach to poverty utilizes income or consumption expenditure (Y_i) as the indicator of well-being, identifies the poor as those with insufficient income to attain minimum basic needs (z), and aggregates their shortfall to a minimum level into a poverty index (Sen, 1976). The poverty headcount, poverty gap, and severity of poverty are the most common indices used in the literature, all belonging to the family of Foster-Greer-Thorbecke (FGT) poverty measures (Foster *et al.*, 1984).

If individual i consumes m goods x_{ij}, $j = 1, 2, \dots, M$, his well-being indicator is $Y_i = \sum_{j=1}^{M} r_j x_{ij}$ where r_j is the market price for good j. The poverty line is determined as $z = \sum_{j=1}^{m} r_j x_{ij0}$ where x_{ij0} belongs to the set of basic needs and $m \in M$. The *FGT* index can be expressed alternatively as

$$FGT_\alpha = \frac{1}{n}\sum_{i=1}^{n}\left[\max\left(\frac{z-Y_i}{z};0\right)\right]^\alpha \tag{1.1}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(1-\frac{y_i}{z}\right)^\alpha L(y_i \le z) \tag{1.2}$$

$$= \frac{1}{n}\sum_{y_i \le z}\left(1-\frac{y_i}{z}\right)^\alpha \tag{1.3}$$

where L is an indicator function and α is a parameter indicating the sensitivity of the index to the distribution among poor – the higher its value, the more sensitive. For $\alpha = 0$, *FGT* is the headcount, for $\alpha = 1$, it is the poverty gap, and for $\alpha = 2$, it represents the severity of poverty.

For decades, many scholars favored a multidimensional perspective to poverty where 'human deprivation is visualized not through income as an intermediary of basic needs but in terms of shortfalls from the minimum levels of basic needs themselves' (Tsui 2002: 70). The latter voices a common argument against the traditional income method on two main grounds. The first questions the assumption of the existence of known prices and markets for all relevant determining deprivation. And even if market prices were to exist, one can challenge the view that these are somehow 'right' in themselves. Instead, they can be seen as equally arbitrary as any other weights chosen by the user (Tsui, 2002). In truth, the latter have the advantage that they allow for a clear understanding of the effects of the weighting scheme.

More interestingly, the monetary approach relies on the implicit assumption of perfect substitutability between attributes, probably too strong an assumption to make. In effect, for poverty or deprivation analysis, some would argue that each attribute is to be considered 'essential' in the sense that a person who does not achieve a minimum threshold in one dimension should be seen as poor, irrespectively of how much he has of the other attributes (Tsui, 2002; Bourguignon and Chakravarty, 2003). In this view of things, substitution between two attributes is only relevant for individuals who are below the minimum level in all dimensions. The idea of essentiality of attributes is consistent with the *union* approach of poverty (Atkinson, 2003; Duclos *et al.*, 2003) and is expressed through the **strong poverty focus axiom**, see below. We will argue that one should also accept an *intermediate* position which allows for substitution between attributes – up to a certain extent – even if some are above the threshold. We can reflect this intermediate view in a **weak version of the poverty focus axiom** which is satisfied by some of information theory indices proposed below.

Based on a critical appraisal of the 'market price approach', and given the strong poverty focus, Tsui derives a set of multidimensional poverty measures following an axiomatic approach, similar in spirit to his work on multidimensional inequality (Tsui, 1995, 1999). Specifically, Tsui extends standard univariate axioms of unidimensional poverty indices, while presenting new axioms tailored to the multivariate poverty context.

Consider the $1 \times m$ vector z of poverty lines for the m attributes. Let us define *multidimensional poverty index* as a mapping from the matrix X and the vector z to a real valued number in the real space $\mathbb{R}$.

$$P(X,z) = G[f(x_{i1}, x_{i2}, \ldots, x_{im}); z] : M(n) \rightarrow \mathbb{R} \tag{1.4}$$

Axioms are imposed on the poverty index $P(X; z)$ directly, rather than to some social evaluation function (as in Tsui, 1999) but these properties will constrain the family of individual functions $f(x)$ and aggregate function $G(.)$. The standard basic set of axioms include: continuity, symmetry, replication invariance, monotonicity, subgroup consistency and ratio-scale invariance.[3] These axioms will restrict the $G(.)$ to be increasing and continuous and the $f(.)$ to be continuous and non-increasing in attributes.

Basic axioms should be complemented with poverty specific properties, which we define below. In this paper we will discuss in depth the implication of the first of these (*poverty focus*). Other axioms are presented as in Tsui (2002).

Axiom 1.1. Strong poverty focus. If any attribute x_{ij} changes such that $x_{ij} \geqslant z_j$ before and after the changes, $P(X; z)$ does not change. This property leads us to not only ignore *individuals above* the poverty minimum threshold in all relevant attributes, but also *attributes above* the minimum level of individuals who do not achieve the minimum in other attributes. Alternatively, **Weak poverty focus** makes the poverty index independent of the attribute levels of non-poor individuals only (Bourguignon and Chakravarty, 2003). In other words, some interplay between attributes above and below the poverty threshold is allowed. Tsui does not consider this weaker version.

Axiom 1.2. Poverty criteria invariance. If $z \neq z'$ then $P(X; z) \leqslant P(Y; z) \Leftrightarrow P(X; z') \leqslant P(Y; z')$ whenever $X(z) = X(z')$ and $Y(z) = Y(z')$. This axiom ensures that there is no dramatic change in the evaluation of poverty for changes in the poverty threshold not affecting the number of poor. In other words, the ordering of distributions does not change, even if the measurement itself might change.

Axiom 1.3. Poverty non-increasing minimal transfer with respect to a majorization criteria.[4] $P(Y; z) \leqslant P(X; z)$ where $Y = BX$ and B is a bistochastic matrix or Pigou-Dalton transfer matrix, and the transfer is among the poor. In order words, the poverty index must be sensitive to the dispersion of the attributes among the poor, which restricts $f(.)$ to be convex.

Define 'basic-rearrangements increasing transfer' as a transfer between individuals p and q such that the resulting distribution has the same marginal in attributes but higher correlation between them.

Axiom 1.4. Poverty non-decreasing rearrangement. If Y is derived from X by a finite sequence of basic-rearrangements increasing transfers among the poor with no one becoming non-poor due to the transfer, then $P(X; z) \leqslant P(Y; z)$. In other words, more correlation between attributes among the poor increases (or leaves unchanged) the measurement of poverty.

The last axiom restricts $f(.)$ to be L-superadditive or, if differentiable, its cross-partial derivatives with respect to attributes must be non-negative, that is, $\frac{\partial f^2}{\partial x_{il}\,\partial x_{im}} \geq 0$.

The resulting multidimensional poverty measures are

$$P_1(X;z) = \frac{1}{n}\sum_{i=1}^{n}\left[\prod_{j=1}^{m} \ln\left(\frac{z_j}{\min(x_{ij};z_j)}\right)^{\delta_j} - 1\right] \tag{1.5}$$

with $\delta_j \geqslant 0$, $j = 1, 2, \ldots, m$, and chosen to maintain convexity of the functions, and

$$P_2(X;z) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m}\delta_j \ln\left(\frac{z_j}{\min(x_{ij};z_j)}\right) \tag{1.6}$$

with $\delta_j \geqslant 0$, $j = 1, 2, \ldots, m$

To better understand the difference between Tsui's poverty index and the traditional income poverty measure we disentangle the index into the implicit individual poverty or shortfall function, and the aggregator function across individuals (or poverty index).

The implicit individual poverty function may be expressed as:

$$p_i = \prod_{j=1}^{m} \ln\left(\frac{z_j}{\min(x_{ij}; z_j)}\right)^{\delta_j} - 1 \qquad (1.7)$$

or

$$p_i = \sum_{j=1}^{m} \delta_j \ln\left(\frac{z_j}{\min(x_{ij}; z_j)}\right) \qquad (1.8)$$

Notice that $p_i = 0$ for those who are above the poverty line in *all* dimensions. We can think of δ_j as the contribution that the relative shortfall in attribute j makes to the individual poverty.

The implicit poverty index is:

$$P(X; z) = \frac{1}{n}\sum_{i=1}^{n} p_i \qquad (1.9)$$

In other words, the FGT version chosen is the poverty gap, which is the first moment of the discrete (empirical) distribution of p_i.

In a closely related paper, Bourguignon and Chakravarty (2003) impose similar axioms to Tsui except for two of them, and present a distinct family of multidimensional poverty indices. Specifically, they replace the subgroup consistency with separability axiom, and allow for correlation increasing transfer to have either an increasing or decreasing effect on the evaluation of poverty depending on the nature of the attributes involved. In other words, they accept both 'Poverty non-decreasing rearrangement' and 'Poverty non-increasing rearrangement'.

The resulting poverty index is of the following general CES-like form:

$$P_\theta(X; z) = \frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{m} w_j \left(\max\left(1 - \frac{x_{ij}}{z_j}; 0\right)\right)^{\theta}\right]^{\alpha/\theta} \qquad (1.10)$$

Disentangling its components, we observe that the implicit individual poverty function or 'shortfall from threshold levels' is:

$$P_\theta(X; z) = \left[\sum_{j=1}^{m} w_j \left(\max\left(1 - \frac{x_{ij}}{z_j}; 0\right)\right)^{\theta}\right]^{1/\theta} \qquad (1.11)$$

where parameters are set so that p_i is increasing and convex. w_j are positive weights attached to each j attribute, whereas θ sets the level of substitutability between shortfalls; the higher the θ, the lower the degree of substitutability. Interesting special cases are, when θ tends to infinity relative deprivations are non-substitutes; and when $\theta = 1$ shortfalls are perfect substitutes. Under both situations, poverty will be defined unidimensionally, in the first case by the attribute deprivation with the highest value, in the second, as a simple weighted sum of attributes. Note that the second option shares some resemblances with the standard income poverty approach whenever the weights are determined using market prices. Convexity of attributes – that is concavity in the space of deprivations – will restrict the parameter to be $\theta \geqslant 1$.

The implicit Poverty index is the α moment of the p_i distribution:

$$P(X;z) = \frac{1}{n}\sum_{i=1}^{n}(p_i)^{\alpha} = FGT_{\alpha} \tag{1.12}$$

The Bourguignon and Chakravarty proposal has the advantage of making explicit the role of the parameters involved in the measure, such as weights, substitution levels between attributes, and a parameter related to the weight to be attached to poverty gaps at different levels of the distribution. Interestingly, the effect of increasing correlation on the poverty index is dependent on the specific relation between the parameters θ and α. The poverty measure is also broader than Tsui's in allowing for a more general formulation of the welfare function G[.] across individuals.[5]

1.3 An information-theoretic analysis of the aggregation functions and poverty measures

The issue of aggregation of attributes in many dimensions has an information-theoretic interpretation and solution which reveals the information content of each poverty aggregator function. In the context of multidimensional measurement of inequality, Maasoumi (1986) proposed functionals for p_i (f(.) in (1.4) above) which would summarize the information in all the attributes in an efficient manner. This 'efficiency' refers to completeness of information being incorporated in any summary or aggregate function. As has been noted above, poverty measures are (moment) functions of the distribution of p_i, $i = 1, 2, \ldots, n$. Every attribute j has a distribution as well, $x_j = (x_{1j}, x_{2j}, \ldots, x_{nj})$. Naturally, the distribution of p_i is derived from, and follows the m distributions x_j, $j = 1, 2, \ldots, m$. In objective, empirical science, the distribution of a variable contains all the information about that variable that is or can be accessed and inferred objectively. Given this truism, one must select functional forms for the aggregator functions p_i that would make its distribution the closest to the distributions of its constituent members, x_js. This ideal can be achieved by solving an information theory inverse problem, based on distributional divergences or distances, which produces 'optimal' functions for p_i.

The basic measure of divergence between two distributions is the difference between their entropies, or the so called *relative entropy*. Let S_i denote the summary or aggregate function for individual i, based on his/her m attributes $(x_{i1}, x_{i2}, \ldots, x_{im})$. Then consider a weighted average of the relative entropy divergences between $(S_1, S_2, \ldots, S_n)$ and each $x_j = (x_{1j}, x_{2j}, \ldots, x_{nj})$, as follows:

$$D_\theta(S, X; w) = \sum_{j=1}^{m} w_j \left[\sum_{i=1}^{n} S_i \left[\frac{(S_i/x_{ij})^{-\theta} - 1}{\theta(\theta - 1)} \right] \right] \tag{1.13}$$

where w_j is the weight attached to the Generalized Entropy divergence from each attribute. Minimizing $D_\theta(.)$ with respect to S_i such that $\sum S_i, = 1$ produces the following 'optimal' Information Theory (IT) aggregation functions:

$$S_i \propto \left(\sum_{j=1}^{m} w_j x_{ij}^{\theta} \right)^{1/\theta} \qquad \text{when } \theta \neq 0 \tag{1.14}$$

$$S_i \propto \prod_{j=1}^{m} (x_{ij})^{w_j} \qquad \text{when } \theta = 0 \tag{1.15}$$

The function $D_\theta(.)$ is linear in the mutual divergences since it is merely a weighted sum or average. One could just as easily consider hyperbolic means of the mutual divergences. Also, the solution functions will be the same if we considered normalized attributes, such as x_{ij}/μ_j where $\mu_j = E(x_j)$ or $x_{ij}/\sum_{i=1} x_{ij}$ which are the attribute shares (see Maasoumi, 1986). Note that the standard consumer theory requirement of convexity of indifference curves in the attribute space will demand θ to be less than or equal to one. In the context of poverty indices, one might consider the relative deprivation functions, $q_{ij} = 1 - x_{ij}/z_j$, in place of x_{ij}. In this case, the convexity requirement is the opposite $\theta \geqslant 1$. See below for this alternative.

We will show here that both Tsui and Bourguignon-Chakravarty indices can be included within one of two approaches to IT indices of poverty. And, as such, these satisfy the axioms advocated by them, as well as being based on aggregator functions which are information efficient, based either on the attribute quantity possessed or on relative poverty gaps ($q_{ij} = 1 - x_{ij}/z_j$). But the IT approach opens the way to more general measures of poverty, including more complex moments than the average/mean functions $\left(\frac{1}{n}\sum_{i=1}^{n}\right)$ favoured in the axiomatic approach.

Another point worth emphasizing is that the first version of IT indices is not limited to observing the strong focus axiom. This means that our indices can allow for substitution, that is, compensation, from an attribute that exceeds its poverty level to another that falls short of it. The individual does not have to be poor in all dimensions to be either found to be poor or non-poor in the multidimensioned context. We think that weak focus is, indeed, a very attractive feature of multidimensional approach which deserves to be examined in many real life situations.

In the empirical part we compare these different approaches for the same data and case study, for a range of substitution parameters and weights.

Aggregate poverty line approach to IT indices of poverty

Case A. Let us define an *aggregate poverty line* S_z that is consistent with the IT aggregator functions S_i derived above:

$$S_z = \left(\sum_{j=1}^{m} w_j z_j^{\theta} \right)^{1/\theta} \qquad \text{when } \theta \neq 0 \tag{1.16}$$

and the generalized geometric mean for $\theta = 0$.

A two-step approach is to:

1. Define the multiattribute relative deprivation function as

$$p_i = \max\left(\frac{S_z - S_i}{S_z}; 0 \right) = \max\left(1 - \frac{S_i}{S_z}; 0 \right) \tag{1.17}$$

2. Define the following IT multi-attribute poverty measures:

$$P_{\alpha}(S;z) = \frac{1}{n} \sum_{i=1}^{n} \left[\max\left(1 - \frac{S_i}{S_z}; 0 \right) \right]^{\alpha} = \frac{1}{n} \sum_{i=1}^{n} p_i^{\alpha} \tag{1.18}$$

This is the αth moment FGT poverty index based on the distribution of $S = (S_1, S_2, \ldots, S_n)$.

Each attribute's poverty line, z_j, plays a role in defining a multiattribute poverty line, S_z, which incorporates the same weights for, and relationship between, the attributes as considered for each individual/unit. All of the axioms which support FGT are applied to individual summary functions of well-being, S_i. All other univariate poverty indices are applicable to the summary distribution.

Notice that the above general formulation allows for the possibility of some substitution between attributes above and below the poverty thresholds provided the individual is poor in at least one dimension. This will be consistent with the weak poverty focus axiom.

If, instead, one prefers to highlight the essentiality of each component and support a strong version of the focus axiom (*union approach*), one has only to replace x_{ij} by the expression $\min(x_{ij}, z_j)$.[6,7] In fact, when $\theta = 0$, and for $w_j = -\delta_j$, the implicit p_i in (1.18) is equivalent to Tsui's individual poverty function. In general, as presented, our measure is non-negative and normalized to be less than one. Tsui's P_1 index is also non-negative but unbounded. This has the disadvantage that the upper bound is dependent on values and units chosen for each poverty line z_j. One interpretation is that our IT measures include a normalized version of Tsui's when $\theta = 0$.

Case B. A similar but somewhat different version of this approach may also be considered. Consider following as described above, but without the consistent derivation of the S_z. Suppose a multidimensional poverty line is chosen directly from the distribution $S = (S_1, S_2, \dots, S_m)$, as though it were a target univariate distribution. Suitable candidates for this line would be the so-called relative poverty lines, such as the lower quantiles, or a percentage of the median of the distribution. Indeed, this has been suggested by D'Ambrosio *et al.* (2004), and Miceli (1997) who seems to have been the first to apply the Maasoumi (1986) approach to poverty, with application to Swiss data.

Component poverty line approach to IT indices of poverty

Consider obtaining summary functions of $q_{ij} = 1 - x_{ij}/z_j$ in place of x_{ij}. q_{ij} can be interpreted as *shortfalls to threshold*, as in Bourguignon and Chakravarty, where for poor persons $0 \leqslant q_{ij} \leqslant 1$ and 'rich' $q_{ij} \leqslant 0$. The optimal IT functionals will be the same as given above. Then the second two-step IT indices of poverty are similarly derived as follows:

1. Let the relative deprivation function be

$$S_{q_i} = \left(\sum_{j=1}^{m} w_j q_{ij}^{\theta} \right)^{1/\theta} \quad \text{when } \theta \neq 0 \quad \text{and} \quad \text{for all } j,\, q_{ij} \geqslant 0 \ \text{ or } \ X_{ij} \leqslant Z_j \tag{1.19}$$

so that the individual poverty function is

$$p_i = \left(\sum_{j=1}^{m} w_j q_{ij}^{\theta} \right)^{1/\theta} \quad \text{and for all } j, q_{ij} \geqslant 0 \text{ or } X_{ij} \leqslant Z_j \tag{1.20}$$

$$= \left(\sum_{j=1}^{m} w_j \ \max(q_{ij};0)^{\theta} \right)^{1} \tag{1.21}$$

In other words, the strong focus axiom and union definition of poverty are imposed. This step obtains an aggregate of relative deprivations which allocates weights to each, and allows trade offs between these relative deprivations in various attributes. Again, this is only for attributes that are below the poverty threshold. The weak focus poverty axiom is not invoked in the second IT approach.[8]

2. Define the multiattribute poverty measure

$$P(S_q; z) = \frac{1}{n} \sum_{i=1}^{n} (S_{q_i})^{\alpha} \tag{1.22}$$

This is the αth moment of the distribution of $S_q = (S_{q1}, S_{q2}, \dots, S_{qn})$.

Here there is no explicit aggregate poverty line. To be explicit, the second IT approach index for two dimensions, and for someone who is poor in both dimensions is as follows:

$$p_i = [w_1(1-(x_{i1}/z_1)^{\theta} + w_1(1-(x_{i2}/z_2)^{\theta}]^{\alpha/\theta} \tag{1.23}$$

which is the same as the Bourguignon-Chakravarty poverty index.

1.4 Empirical results

This section presents an application of the proposed poverty measures to data from Indonesia. The exercise highlights the inevitability of making value judgments when comparing any two multivariate distributions.

We compare three-dimensional distributions of Indonesians' expenditure, health status, and level of education according to the three largest ethnic groups. These are Jawa, Sunda, and Betawi, which contain 52, 18 and 5 per cent of the total Indonesian population, respectively.[9] The exercise is meant to be merely illustrative and, for this reason, we choose to represent well-being by only three attributes. Naturally, results can be extended to more dimensions. The choice of dimensions was made given the wide agreement on their fundamental role as both means and ends – particularly in the case of education and health (Anand and Sen, 2000).

Data come from the 2000 Indonesian Family Life Survey (IFLS) conducted by RAND, UCLA and the Demographic Institute of the University of Indonesia. The IFLS is a continuing longitudinal socioeconomic and health survey, representing 83 per cent of the Indonesian population living in 13 provinces (out of 26). It collects data on individual respondents, their families, their households, the communities in which they live, and the health and education facilities they use (Strauss *et al.*, 2004). The IFLS was previously conducted in 1993, 1997, and 1998, but data on health status are publicly available only for 2000.

Approximately 10,400 households and 39,000 individuals were interviewed in 2000. We will restrict the study to individuals with complete information on all relevant variables, omitting just over one per cent of the sample.

The indicators used are *real per capita expenditure, level of hemoglobin, and years of education achieved by the head of household*. Nominal per capita expenditure data is adjusted using a temporal deflator (Tornquist CPI, base year December 2000) and a spatial deflator (regional poverty lines) (Strauss *et al.*, 2004). Individuals' hemoglobin levels are expressed in grams per deciliter (g/dl). Low levels of hemoglobin indicate deficiency of iron in the blood where '…[i]ron deficiency is thought to be the most common nutritional deficiency in the world today' (Thomas *et al.*, 2003: 4).[10] Given that normal values of hemoglobin depend on sex and age, we adjusted individual values to transform them into equivalent adult levels.[11] Tables 1.A1 and 1.A2 in the appendix present basic statistics for these variables, including correlation coefficients between them.

Computing poverty involves choosing a cut-off point for each indicator. To allow for sensitivity to different poverty lines we use two values representing reasonable boundaries for alternative thresholds. These can also be related to extreme poverty and poverty lines, as in the traditional poverty literature. In particular, for per capita expenditure we utilise Strauss *et al*. (2004)'s values of Rp. 100,000 and Rp. 150,000, respectively;[12] for hemoglobin 12 g/dl and 13 g/dl;[13] and for education 4 and 6 years of schooling.[14]

Table 1.1 presents measurements of poverty for each attribute, using the FGT index for values of $\alpha = [0, 1, 2]$. Interestingly enough, the ordering of groups differs for each dimension. In particular, the poorest group in expenditure (Jawa) is in the second position in health and education, whereas the poorest in terms of education outcomes (Betawi) is the richest both in expenditure and health outcomes. The Sunda group, on the other hand, has the highest poverty measurement in health, the second highest in expenditure, and the lowest in education.[15] In this context, the decision on how to aggregate the different dimensions across ethnic groups becomes particularly relevant.[16]

Employing multidimensional poverty indices involves, necessarily, a significant loss of information. Depending on how the aggregation is done – in terms of functional form, indicator variables, and parameter values – the results will vary in terms of cardinal values and, in some cases, the ordinal rankings of the distributions. Figures 1.1 to 1.6 show the resulting measurements (y-axis) using the two approaches presented in the previous section, as the parameter of substitutability between attributes (θ) varies (x-axis). We utilize equal weighting scheme (that is $w_j = 1/3$ for all j) and allow α to correspond to poverty headcount formula in the left panel while severity of poverty in the right panel (Figures 1.7 to 1.9 present a

Table 1.1 Univariate poverty measurement by regions. Indonesia, 2000

	Ethnic groups								
	Jawa			*Sunda*			*Betawi*		
Extreme poverty	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
Expenditure	0.141	0.032	0.011	0.135*	0.029*	0.011*	0.072	0.018	0.007
Hemoglobin	0.109	0.011*	0.003*	0.127	0.012	0.002	0.101	0.009*	0.002*
Education	0.320*	0.209	0.156	0.234	0.149	0.109	0.344	0.234	0.178
Poverty	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
Expenditure	0.344	0.102	0.042	0.323*	0.097*	0.040*	0.262	0.068	0.026
Hemoglobin	0.247*	0.024*	0.005*	0.273	0.026	0.005	0.261*	0.022*	0.004*
Education	0.404	0.269	0.204	0.318	0.201	0.148	0.455	0.301	0.230

*Not different from the higher value at 95%
Source: Authors' calculations.

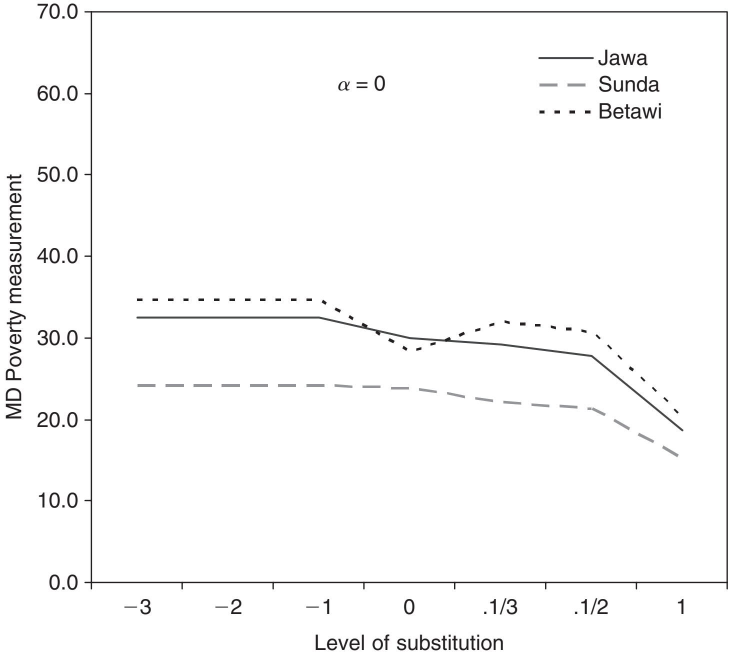

Figure 1.1 Aggregate poverty line approach weak focus extreme poverty, equal weight

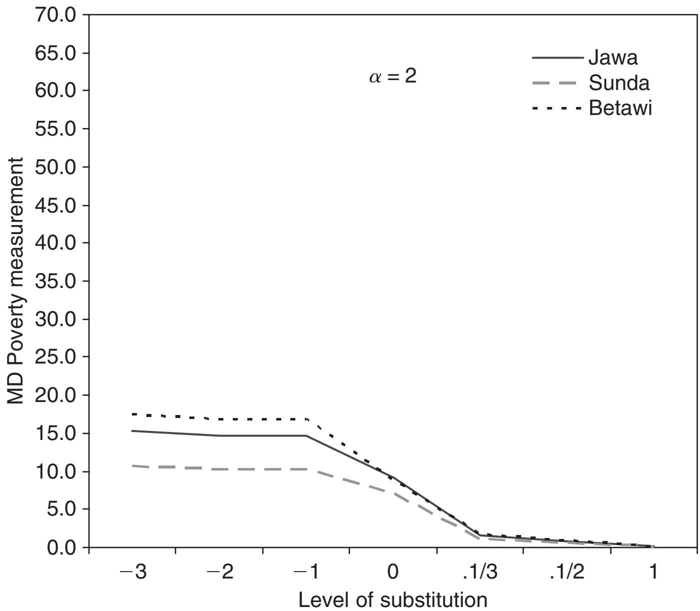

Figure 1.2 Aggregate poverty line approach weak focus extreme poverty, equal weight

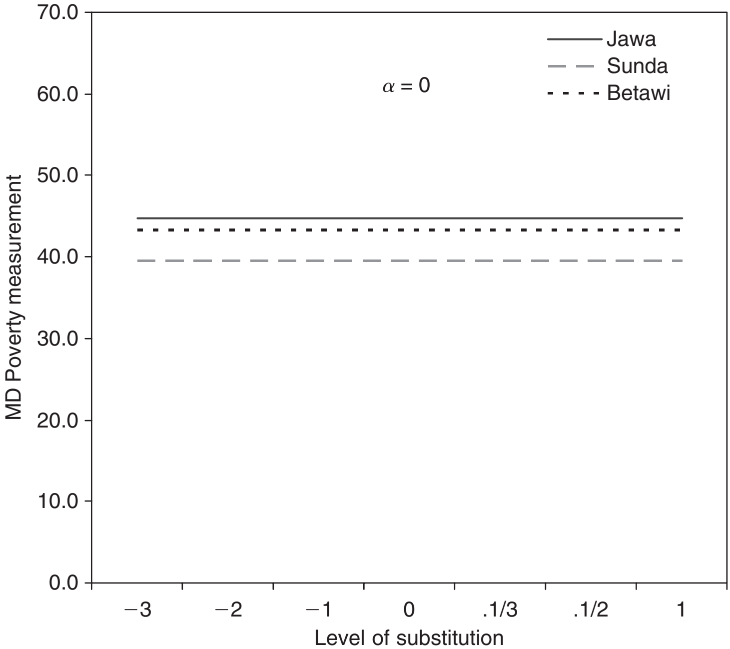

Figure 1.3 Aggregate poverty line approach strong focus extreme poverty, equal weight

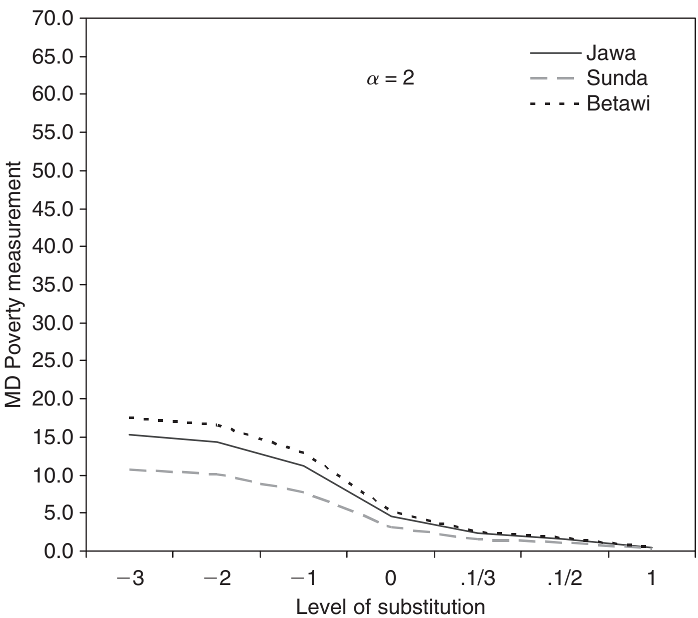

Figure 1.4 Aggregate poverty line approach strong focus extreme poverty, equal weight

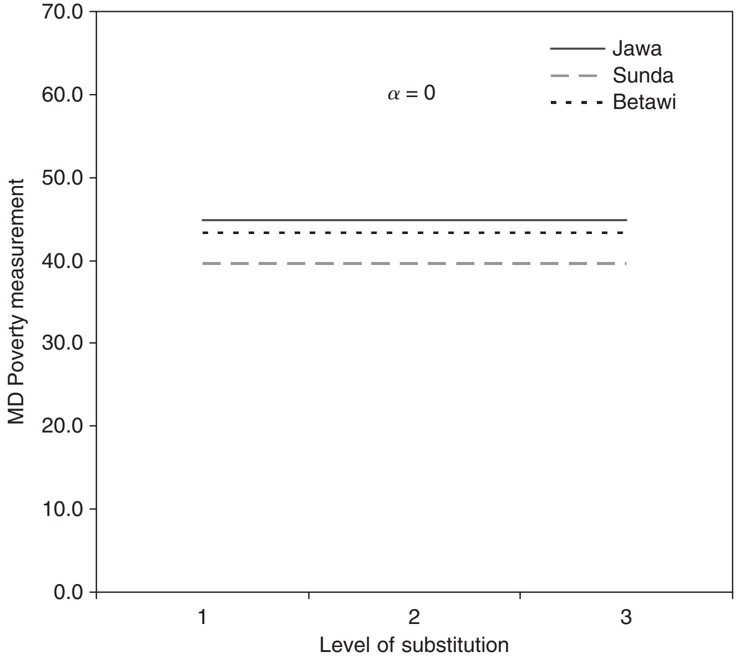

Figure 1.5 Component poverty line approach strong focus extreme poverty, equal weight

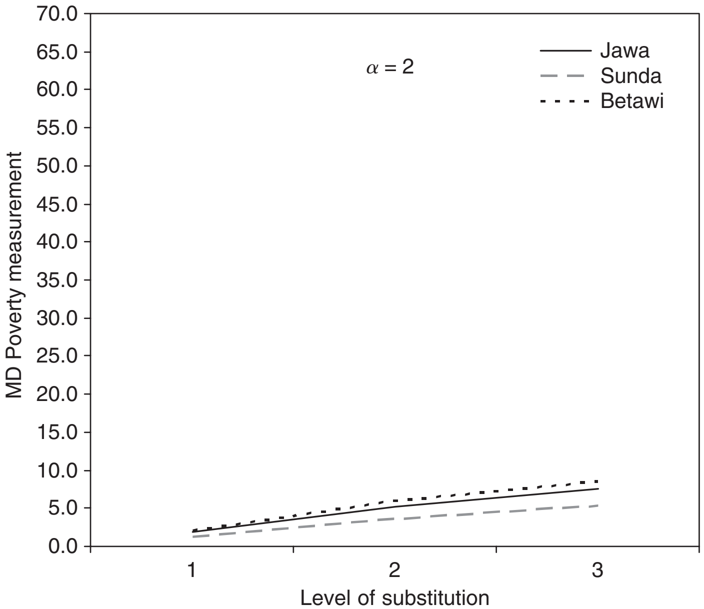

Figure 1.6 Component poverty line approach strong focus extreme poverty, equal weight

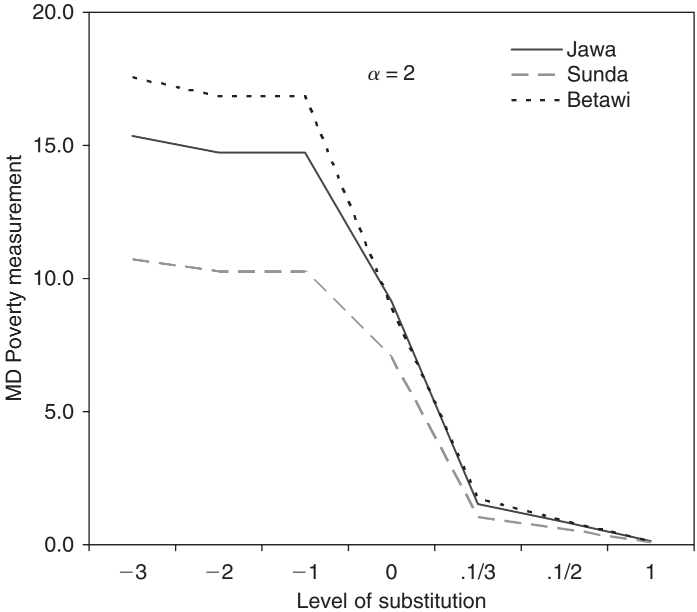

Figure 1.7 Aggregate poverty line approach weak focus extreme poverty, equal weight. Magnified version of Figure 1.2

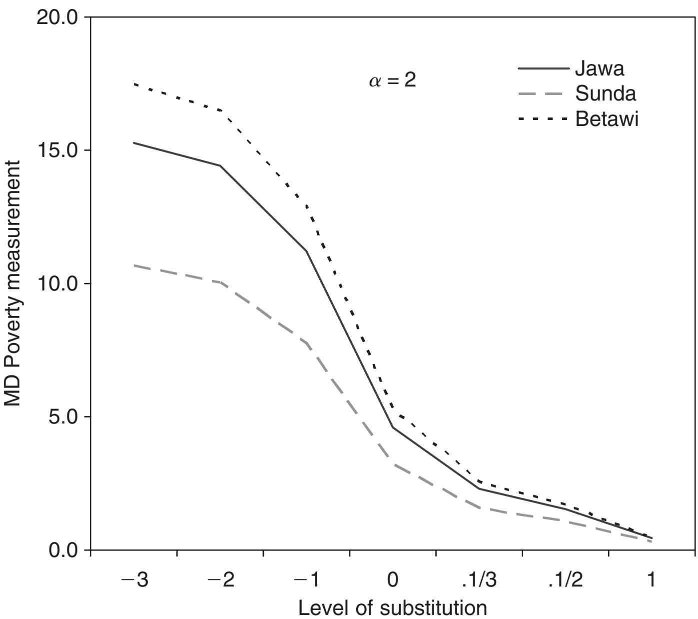

Figure 1.8 Aggregate poverty line approach strong focus extreme poverty, equal weight. Magnified version of Figure 1.4

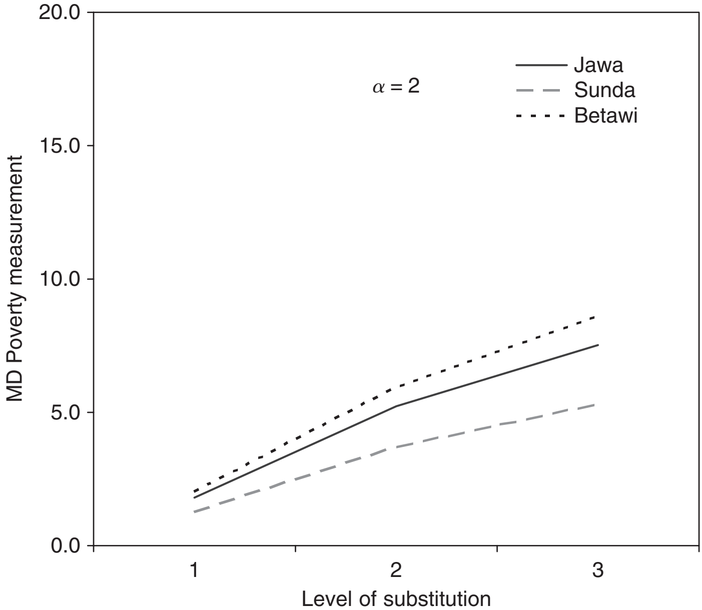

Figure 1.9 Component poverty line approach strong focus extreme poverty, equal weight. Magnified version of Figure 1.4

magnified version of graphs on the right). Table 1.A3 in the appendix includes these poverty measurements and, for sensitivity analysis we add alternative poverty lines, weighting schemes and poverty gap measures. We also include CDFs for each combination of aggregate well-being function computed for the measurements (Figures 1.A2 to 1.A4).

We first compare the results with those obtained from the univariate poverty analysis. The Sunda group, which is ranked first, second and third in each distinct dimension in the multivariate analysis, becomes unambiguously the better-off ethnic group. This is true for all combinations of approaches and parameter values calculated here.

The comparison between the Jawa and Betawi population is less straightforward. Poverty headcount measurements ($\alpha = 0$) provide unambiguous rankings for all three figures. Still, Betawi has higher poverty values when using the first approach and weak focus while Jawa is placed first when strong focus is invoked. This might reflect the fact that within Jawas, low levels of expenditure are accompanied by relatively high education outcomes – relative to the Betawi group.

When the distribution within the poor is considered – that is, when α is higher than 0 – the ordering of groups depends on the choice of level of substitutability between attributes. In particular, we find unambiguous rankings for lower θ, but the distinction between groups vanishes when θ is allowed to be positive.[17] All these results are robust to the two weighting strategies employed here. We expect that only very extreme a priori weight systems may produce results that are closer to the unidimensional poverty values.

Notice that, as expected, the measured poverty rates increase as the substitutability between attributes decreases. At the extreme, when there is no substitution, multidimensional poverty rates will equal the unidimensional poverty rate for the component of the index with the highest poverty. For all Indonesian regions this is education. Recall that higher substitution between attributes corresponds to *high* values of θ in the first IT approach and to *low* values of θ in the component poverty line approach (based on shortfalls).

Finally, within the Aggregate Poverty Line approach we can observe the implications of using the weak versus the strong poverty focus axiom. For each combination of (w_j, θ, α) the weak poverty focus consistent measures yield lower measurements than those consistent with strong focus. This is due to the fact that the former allows for some degree of substitution (compensation) between attributes for those who are poor in one dimension and not in some other such that they end up being above the multidimensional poverty threshold. This example shows that employing the weak poverty focus axiom can be seen as an intermediate case between union and intersection approaches.

1.5 Conclusions

We have presented the Information Theory approach to multidimensional poverty measurement in a connected way that allows both new measures and a deeper interpretation of the existing methods, primarily based on the axiomatic approaches. The IT approach emphasizes clarity in aggregation choices that, it is argued, are inevitable in any multidimensional setting. The univariate methods are not exempt from this. By making aggregation issues explicit, the IT methods are also able to reveal the meaning and the working of the multidimensional context when one allows compensation to an individual/household from the above threshold attributes for those attributes that fall short. We feel it is essential to have an accommodation for this possibility since, otherwise, the case for a multidimensional approach to poverty and welfare may not exceed far beyond adding up, or averaging, over many dimensions. Future work will focus on differential substitution levels between individual categories, and attribute levels. These nonlinearities require deeper and careful analysis in each case study and empirical setting.

We have shown where, and under which conditions, our IT measures are identical to the index families proposed earlier in the literature, and have new IT indices when some of those conditions are relaxed. The Indonesian case study brings out some of these issues, but not all. The CDF graphs are merely indicative (but not statistically definitive) of a great degree of robustness in our ranking of poverty status of different regions of the country at a particular point of time. Nevertheless, some degree of fragility of numerical conclusions was observed relative to the degree of substitution between attributes, and inequality aversion within the group classified as poor, as well as allowance for compensation from higher-than-threshold attributes. The size of the group which is not poor in all dimensions deserves a deeper examination and may itself characterize economies and societies in meaningful ways. We defer these issues to future research.

Notes

1. We thank the editors for their invitation to participate and for constructive input and reviews. This research was supported by funds from the Robert & Nancy Dedman Chair in Economics at SMU. Finally, we thank Kathleen Beegle and Jed Friedman for providing the adjusted expenditure data.
2. Of course, there are many entropies, including Shannon's which underlies Theil's inequality measures, and Generalized Entropy, which underpins the GE measures of inequality and Atkinson's family. Maasoumi (1993) emphasizes the axiomatic properties that justify different entropies and metrics, which are the same, alas with different names, that support different measures of inequality and poverty.
3. See Tsui (2002) for a definition of axioms in the multidimensional poverty context.
4. This refers to distributional majorization criteria, multidimensional extensions of the Pigou–Dalton Principle, Uniform Majorization or Uniform Pigou-Dalton Majorization, see Kolm (1977).
5. Bourguignon and Chakravarty also present an interesting case where θ depends on the poverty level, so that the substitution between shortfalls changes according to how far the individual is from the poverty line.
6. An intersection approach to poverty could be also obtained if the sample is restricted to individuals with all attributes below their threshold.
7. To clarify the difference between weak and strong versions consider the individual poverty functions when only two attributes are included, and $\theta = 0$. For individuals who are poor in both dimensions, both the weak and the strong version would lead to

$$p_i = 1 - \left(\frac{x_{j1}}{z_1}\right)^{w_1}\left(\frac{x_{j2}}{z_2}\right)^{w_2}$$

   But for persons who are poor only in one dimension – say, x_1 – the weak version would $p_i = \max\left[1 - \left(\frac{x_{j1}}{z_1}\right)^{w_1}\left(\frac{x_{j2}}{z_2}\right)^{w_2};0\right]$ be which will be $p_i = 1 - \left(\frac{x_{j1}}{z_1}\right)^{w_1}\left(\frac{x_{j2}}{z_2}\right)^{w_2}$ 0; r $p_i = 0$ depending on the specific relation between $\left(\frac{x_{j1}}{z_1}\right)^{w_1}$ and $\left(\frac{x_{j2}}{z_2}\right)^{w_2}$. Whereas the strong version will be $p_i = 1 - \left(\frac{x_{i1}}{z_1}\right)^{w_1}$.
8. The reason why weak focus cannot be invoked by the second approach is that $q_{ij} < 0$ when the individual possesses more than the poverty line level of that attribute. For even θ this implies that the farther away (richer) the person is the higher his value of *qij*, that is, his 'deprivation'. This is clearly an undesirable property.
9. We assign to each individual the ethnic group as declared by his or her head of household. The question strictly refers to the influence of ethnicity on daily activities ('Which ethnical group is primarily influential in daily activities of your household?'. Answers are classified in 25 ethnic groups including 'Others').
10. Low levels of haemoglobin are linked to susceptibility to diseases, fatigue, and lower levels of productivity. It reflects the combination of a diet that is high in animal proteins (primary source of iron) and greater absorption capacity (which is reduced by disease insults, presence of worms, loss of blood and diets high on rice). More generally, low levels are related to iron deficiency. See WHO (2001) and Thomas (2001).
11. We use threshold values from the WHO report (2001) to compute the table of equivalence (Table 6, chapter 7). Normal levels of haemoglobin also vary with long exposure to altitudes – which we ignore for our calculations but given our sample of Indonesia in this survey it shouldn't be problematic. Also studies show that in US individuals from

African extraction tend to have normally lower values. A thorough assessment of anaemia for the Indonesian population should consider both issues.

12. See chapter three in Strauss *et al*. (2004). In December 2000, the exchange rate for the Rupiah was Rp. 9,480 per one US dollar.
13. From the WHO report, a male adult is considered anaemic, possibly suffering from iron deficiency, if his haemoglobin level is below 13 g/dl.
14. To avoid later computational problems, we assigned to individuals with no education a value of 0.5 (instead of 0).
15. The previous results should be evaluated in the light of statistical significance of differences as presented in the table.
16. In the Annex we include a table with basic statistics for variables employed.
17. A proper assessment of this conclusion requires the computation of standard errors of multidimensional measurements. We intend to include them in future versions.
18. Expenditure is considered in natural log form.

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Appendix

The following presents basic summary statistics and the figures show their respective distribution, using Kernel approximation.

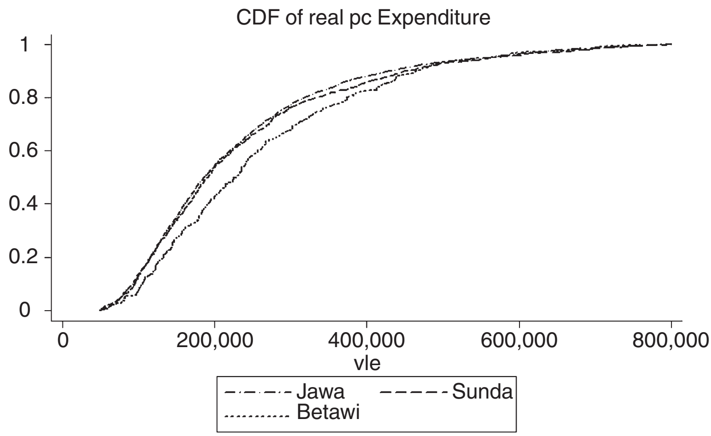
Table 1.A1 Summary statistics by regions: Indonesia, 2000

Variable	***Obs.***	***Mean***	***Std. Dev.***	***Min.***	***Max***
	Jawa				
Real per capita expenditure (Rp.)	17,097	271,347	287,322	20,348	5,236,150
Haemoglobin (g/dl)	17,097	13.95	1.71	3.6	25.8
Education of head of hh	17,089	6.25	4.46	0.5	19.0
	Sudan				
Real per capita expenditure (Rp.)	5,932	294,857	338,738	24,391	6,066,339
Haemoglobin (g/dl)	5,932	13.86	1.71	3.5	19.4
Education of head of hh	5,927	6.78	4.28	0.5	19.0
	Betawi				
Real per capita expenditure (Rp.)	1,576	306,096	316,578	42,577	3,901,813
Haemoglobin (g/dl)	1,576	13.94	1.67	3.1	20.1
Education of head of hh	1,576	6.20	4.57	0.5	17.0

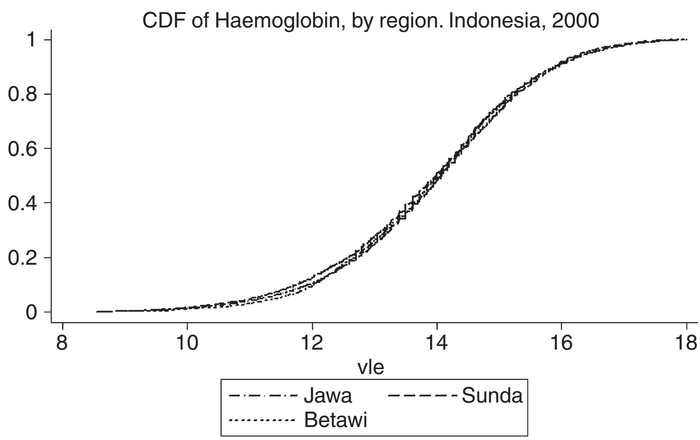
Table 1.A2 Correlation coefficients: Indonesia, 2000

	Expenditure	*Haemoglobin*	*Education*
Pearson Correlation Coefficients (sign 0.05)			
Jawa			
Expenditure	1.0000		
Haemoglobin	0.0675*	1.0000	
Education	0.3354*	0.0688*	1.0000
Sunda			
Expenditure	1.0000		
Haemoglobin	0.0989*	1.0000	
Education	0.3456*	0.1252*	1.0000
Betawi			
Expenditure	1.0000		
Haemoglobin	0.0703*	1.0000	
Education	0.3104*	0.0635*	1.0000
Spearman Correlation Coefficients (sign 0.05)			
Jawa			
Expenditure	1.0000		
Haemoglobin	0.0869*	1.0000	
Education	0.3889*	0.0789*	1.0000
Sunda			
Expenditure	1.0000		
Haemoglobin	0.1277*	1.0000	
Education	0.4327*	0.1119*	1.0000
Betawi			
Expenditure	1.0000		
Haemoglobin	0.1170*	1.0000	
Education	0.3905*	0.0711*	1.0000
Kendall Correlation Coefficients (sign 0.05)			
Jawa			
Expenditure	0.9997		
Haemoglobin	0.0577*	0.9935	
Education	0.2632*	0.0520*	0.8713
Sunda			
Expenditure	0.9991		
Haemoglobin	0.0854*	0.9941	
Education	0.2858*	0.0729*	0.8400
Betawi			
Expenditure	0.9976		
Haemoglobin	0.0780*	0.9935	
Education	0.2616*	0.0470*	0.8832

Source: Authors' calculation from IFL3 2000.



Source: Authors' from IFLS3 2000



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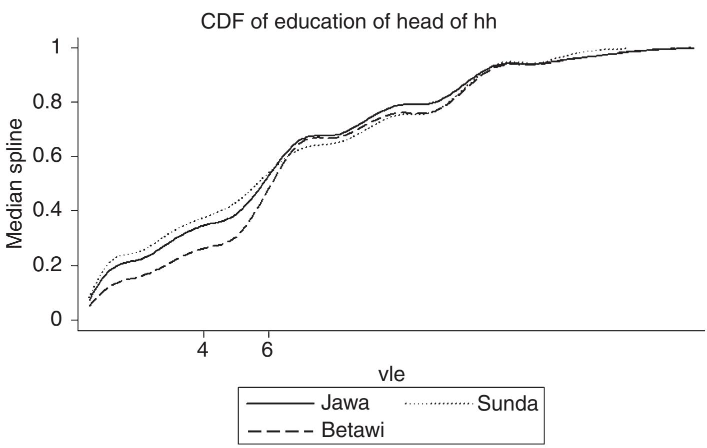


Source: Authors' from IFLS3 2000

Figure 1.A1 CDFs for univariate distributions

The following table presents measurements of multidimensional poverty using the approaches introduced in the chapter.[18] We utilize two weighting schemes (equal weighting and giving half the importance to expenditure), and distinct values for the substitution level θ (from -3 to 1 in the first approach and from 0 to 3 in the second). This is to comply with the convexity requirement in the space of attributes. Finally, we use the three standard α values of FGT measures between 0 and 2. The shading of cells indicates the ranking of the distributions, with the darkest being the highest poverty level in each combination of index and parameters.

Table 1.A3 Multivariate poverty measurement by regions. Indonesia, 2000

EXTREME POVERTY

	Ethnic groups								
	Jawa			*Sunda*			*Betawi*		
IT -Aggregate Poverty Line Approach WEAK FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = - 3	32.48	20.65	15.36	24.20	14.77	10.74	34.65	23.20	17.55
θ = - 2	32.49	20.05	14.71	24.17	14.31	10.26	34.65	22.56	16.84
θ = - 1	32.49	20.05	14.71	24.17	14.31	10.26	34.65	22.56	16.84
θ = 0	29.92	31.32	9.13	23.84	23.37	7.03	28.31	33.68	8.90
θ = 1/3	29.14	5.85	1.52	22.17	4.21	1.06	32.02	6.55	1.70
θ = 1/2	27.82	4.25	0.84	21.37	3.09	0.59	30.76	4.71	0.92
θ = 1	18.74	1.23	0.13	15.29	0.97	0.10	20.17	1.23	0.13
weights {1/2, 1/4, 1/4}									
θ = - 3	32.53	20.54	15.24	24.13	14.68	10.65	34.61	23.08	17.43
θ = - 2	32.50	19.67	14.32	24.13	14.03	9.97	34.61	22.17	16.41
θ = - 1	32.52	16.63	10.95	24.14	11.78	7.54	34.61	18.85	12.59
θ = 0	29.07	30.39	7.14	23.35	22.94	5.49	27.53	32.89	6.89
θ = 1/3	28.01	4.15	0.80	21.49	2.98	0.55	30.73	4.56	0.86
θ = 1/2	26.43	2.88	0.42	20.50	2.08	0.28	29.51	3.09	0.43
θ = 1	15.99	0.76	0.06	13.24	0.59	0.04	15.47	0.69	0.05
IT -Aggregate Poverty Line Approach STRONG FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = - 3	44.81	20.67	15.27	39.59	14.81	10.68	43.30	23.18	17.46
θ = - 2	44.81	19.97	14.42	39.59	14.32	10.05	43.30	22.42	16.50
θ = - 1	44.81	17.35	11.23	39.59	12.42	7.76	43.30	19.50	12.88
θ = 0	44.81	11.19	4.61	39.59	8.07	3.18	43.30	12.51	5.26
θ = 1/3	44.81	8.05	2.29	39.59	5.89	1.59	43.30	8.91	2.58
θ = 1/2	44.81	6.65	1.52	39.59	4.91	1.06	43.30	7.30	1.70
θ = 1	44.81	3.68	0.43	39.59	2.83	0.32	43.30	3.90	0.46
weights {1/2, 1/4, 1/4}									
θ = - 3	44.81	20.80	15.48	39.59	14.89	10.83	43.30	23.34	17.68
θ = - 2	44.81	20.54	15.15	39.59	14.70	10.59	43.30	23.06	17.32
θ = - 1	44.81	19.45	13.77	39.59	13.91	9.58	43.30	21.85	15.76
θ = 0	44.81	9.09	3.06	39.59	6.55	2.10	43.30	10.13	3.48
θ = 1/3	44.81	6.15	1.33	39.59	4.51	0.92	43.30	6.76	1.48
θ = 1/2	44.81	4.99	0.85	39.59	3.70	0.59	43.30	5.43	0.93
θ = 1	44.81	2.70	0.23	39.59	2.09	0.17	43.30	2.81	0.23
IT - Component Poverty Line Approach STRONG FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = 1	44.81	7.38	1.82	39.59	5.43	1.28	43.30	8.11	2.03
θ = 2	44.81	12.53	5.23	39.59	9.20	3.69	43.30	13.88	5.95
θ = 3	44.81	15.04	7.54	39.59	11.04	5.31	43.30	16.66	8.59
weights {1/2, 1/4, 1/4}									
θ = 1	44.81	5.68	1.07	39.59	4.20	0.75	43.30	6.19	1.17
θ = 2	44.81	10.94	3.97	39.59	8.05	2.80	43.30	12.09	4.51
θ = 3	44.81	13.73	6.27	39.59	10.09	4.42	43.30	15.20	7.14

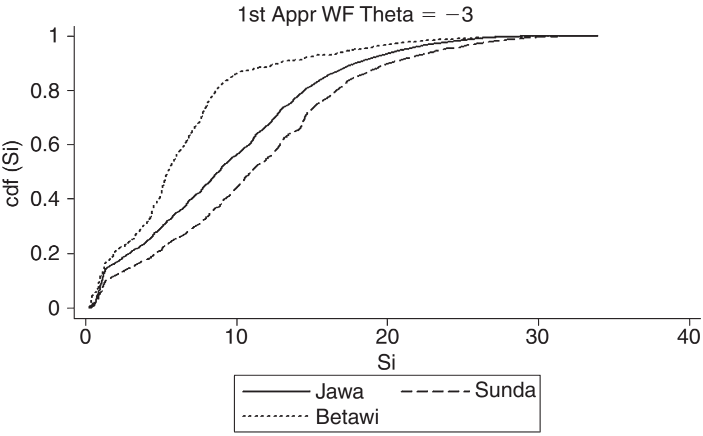
Source: authors' calculation from IFL3 2000.

Table 1.A3 Multivariate poverty measurement by regions. Indonesia, 2000 (Cont.)

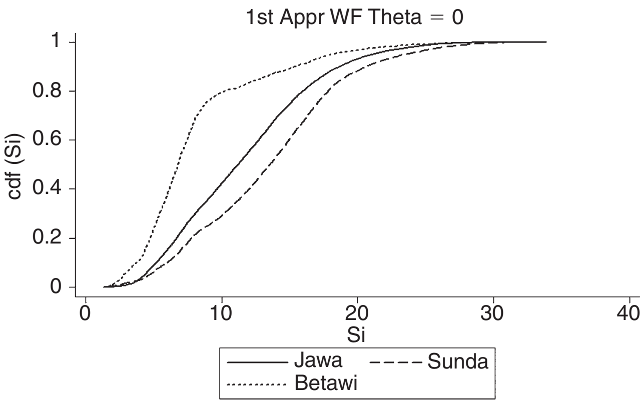
POVERTY

	Ethnic groups								
	Jawa			*Sunda*			*Betawi*		
IT - Aggregate Poverty Line Approach WEAK FOCUS									
	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\theta = -3$	47.00	26.24	19.69	41.98	19.69	14.22	49.60	29.31	22.20
$\theta = -2$	46.81	25.19	18.53	41.60	18.86	13.31	49.61	28.13	20.93
$\theta = -1$	46.75	22.35	15.28	41.54	16.66	10.86	49.42	24.97	17.34
$\theta = 0$	46.25	13.92	6.14	41.01	10.49	4.37	48.33	15.43	6.92
$\theta = 1/3$	45.63	10.12	3.20	40.37	7.79	2.33	47.58	11.10	3.55
$\theta = 1/2$	45.18	8.37	2.18	40.15	6.54	1.61	46.99	9.10	2.39
$\theta = 1$	42.53	4.43	0.67	38.22	3.72	0.54	44.06	4.66	0.69
weights {1/2, 1/4, 1/4}									
$\theta = -3$	47.74	25.89	19.33	42.73	19.40	13.93	50.15	28.91	21.81
$\theta = -2$	47.44	24.43	17.74	42.14	18.23	12.69	50.04	27.29	20.08
$\theta = -1$	47.07	20.73	13.60	41.83	15.37	9.60	49.56	23.18	15.46
$\theta = 0$	45.96	11.26	4.10	41.01	8.50	2.91	48.16	12.41	4.58
$\theta = 1/3$	45.15	7.71	1.89	40.30	5.97	1.37	47.02	8.34	2.05
$\theta = 1/2$	44.57	6.25	1.24	39.94	4.92	0.91	46.49	6.67	1.31
$\theta = 1$	41.51	3.22	0.37	37.47	2.73	0.29	42.72	3.23	0.35
IT - Aggregate Poverty Line Approach STRONG FOCUS									
	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\theta = -3$	65.33	26.25	19.45	60.72	19.76	14.04	64.30	29.28	21.93
$\theta = -2$	65.33	25.03	17.96	60.72	18.82	12.87	64.30	27.90	20.30
$\theta = -1$	65.33	21.69	13.96	60.72	16.28	9.88	64.30	24.18	15.85
$\theta = 0$	65.33	15.25	6.69	60.72	11.57	4.76	64.30	16.88	7.54
$\theta = 1/3$	65.33	11.84	3.82	60.72	9.15	2.75	64.30	12.98	4.25
$\theta = 1/2$	65.33	10.26	2.77	60.72	8.01	2.02	64.30	11.17	3.05
$\theta = 1$	65.33	6.66	1.06	60.72	5.43	0.82	64.30	7.06	1.12
weights {1/2, 1/4, 1/4}									
$\theta = -3$	65.33	26.64	20.06	60.72	20.00	14.51	64.30	29.76	22.60
$\theta = -2$	65.33	26.15	19.42	60.72	19.63	14.01	64.30	29.19	21.91
$\theta = -1$	65.33	24.66	17.52	60.72	18.49	12.55	64.30	27.53	19.82
$\theta = 0$	65.33	12.88	4.67	60.72	9.93	3.35	64.30	14.19	5.24
$\theta = 1/3$	65.33	9.32	2.33	60.72	7.24	1.68	64.30	10.10	2.56
$\theta = 1/2$	65.33	7.97	1.64	60.72	6.27	1.19	64.30	8.55	1.77
$\theta = 1$	65.33	5.12	0.61	60.72	4.22	0.47	64.30	5.30	0.62
IT - Component Poverty Line Approach STRONG FOCUS									
	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\theta = 1$	65.33	10.03	2.56	60.72	7.87	1.89	64.30	10.90	2.79
$\theta = 2$	65.33	16.51	6.92	60.72	12.85	5.07	64.30	18.15	7.74
$\theta = 3$	65.33	19.77	9.93	60.72	15.36	7.26	64.30	21.75	11.13
weights {1/2, 1/4, 1/4}									
$\theta = 1$	65.33	7.88	1.54	60.72	6.23	1.14	64.30	8.44	1.65
$\theta = 2$	65.33	14.47	5.26	60.72	11.30	3.85	64.30	15.85	5.87
$\theta = 3$	65.33	18.08	8.25	60.72	14.08	6.04	64.30	19.86	9.25

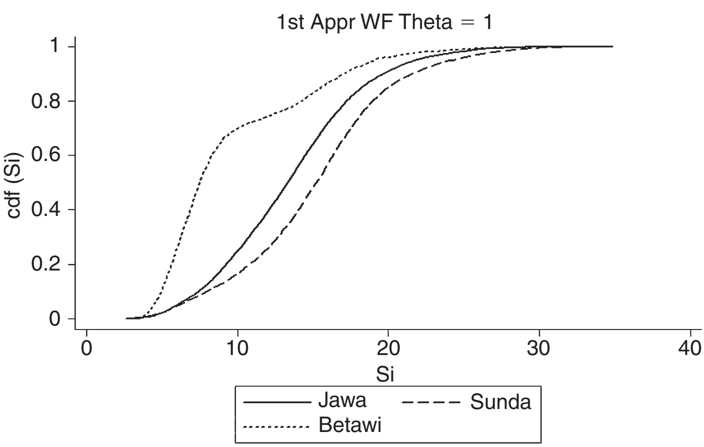
Source: authors' calculation from IFL3 2000.



Source: Authors' calculation from IFLS3 – Equal weight

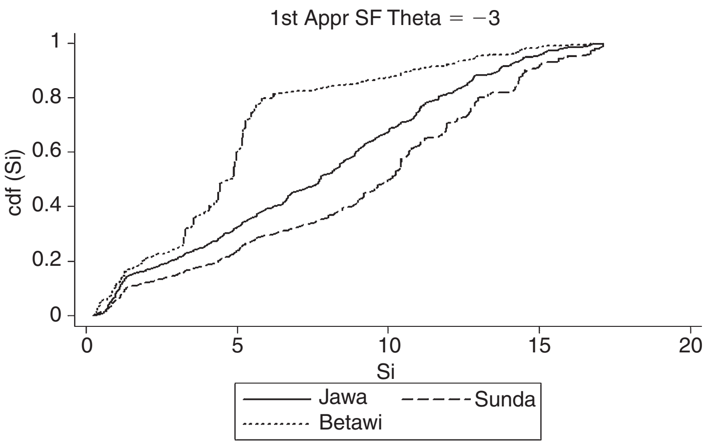


Source: Authors' calculation from IFLS3 – Equal weight

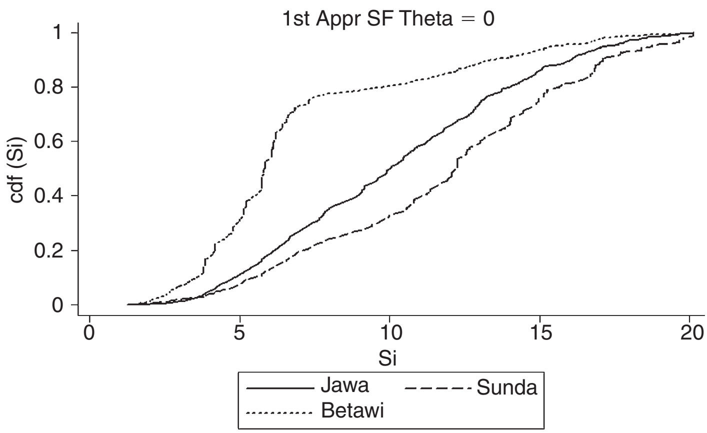


Source: Authors' calculation from IFLS3 – Equal weight

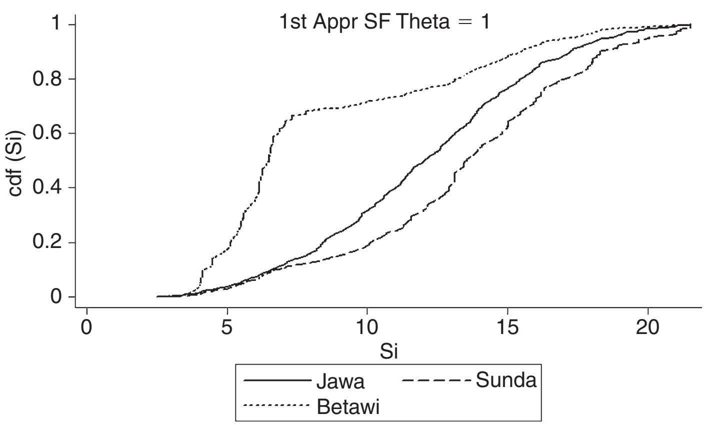
Figure 1.A2 CDFs of aggregated well-being. First Approach (Weak Focus) – Equal weight



Source: Authors' calculation from IFLS3 – Equal weight

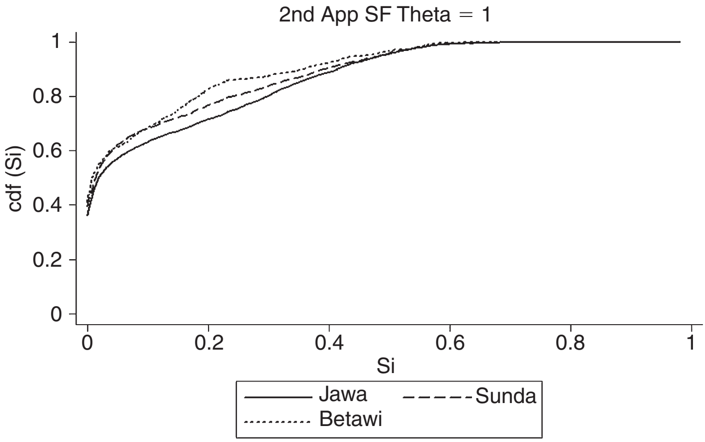


Source: Authors' calculation from IFLS3 – Equal weight

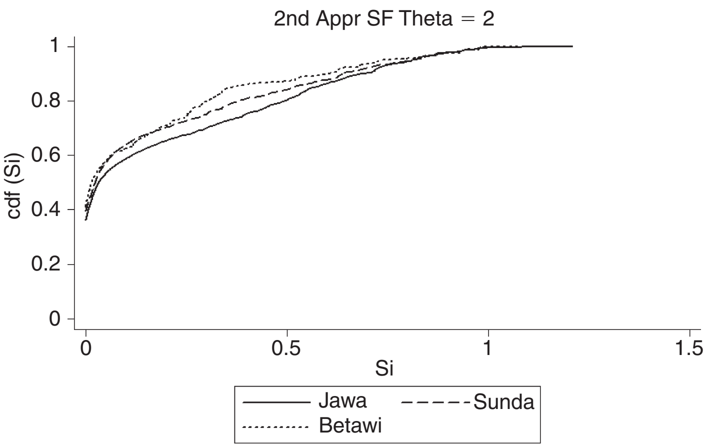


Source: Authors' calculation from IFLS3 – Equal weight

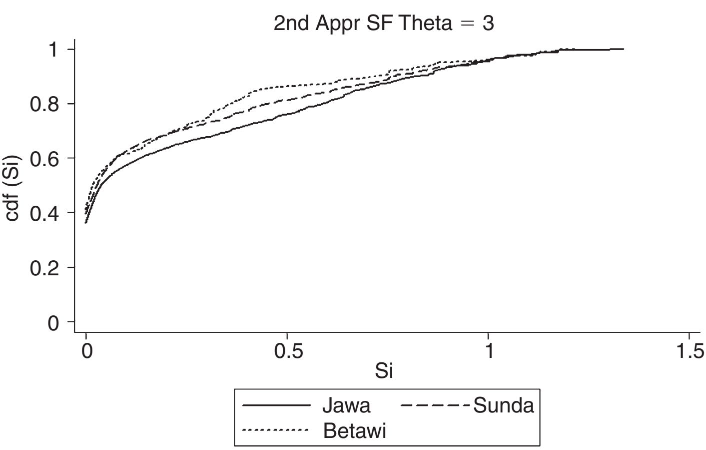
Figure 1.A3 CDFs of aggregated well-being. First Approach (Strong Focus) – Equal weight



Source: Authors' calculation from IFLS3 – Equal weight



Source: Authors' calculation from IFLS3 – Equal weight



Source: Authors' calculation from IFLS3 – Equal weight

Figure 1.A4 CDFs of aggregated well-being. Second Approach (Strong Focus) – Equal weight

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