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Part I

Equity Among Overlapping Generations

1

Pension Contributions and Capital Accumulation*

Toshihiro Ihori

1.1 Introduction

In recent times, much attention has been given to the long-run macroeconomic and intergenerational redistribution effects of public pension reform. It is well recognized that the pay-as-you-go system is not attractive when the rate of population growth is declining in an ageing society. However, the movement from pay-as-you-go financing to full funding is hard in terms of intergenerational equity, just as reducing the public debt–GDP ratio is hard. Researchers have investigated mechanisms under which a decentralized economy might successfully change from a public pay-as-you-go pension scheme to private fully funded schemes. There have been several important attempts to investigate such pension reforms. The standard analysis is of simulation studies using overlapping generations models based on Auerbach and Kotlikoff (1987). Cifuentes and Valdes-Prieto (1997), among others, offer the insightful result from a simulation model that describes the transition in detail, year by year. Mulligan and Sala-i-Martin (1999a,b) present a useful survey of various theories of social security.¹ Hatta and Oguchi (1999) develop the simulation study on Japanese pension reforms; see also Ihori (2002) and Oshio (2004).

Because most of the social security contributions from current workers go directly to fund benefits for current retirees in the unfunded system, the social security system does not significantly increase the level of government savings. Therefore, to the extent that social security reduces private saving, it will also tend to reduce the total level of saving in the economy. However, time-series and cross-country estimates are inconsistent and fraught with conceptual difficulties. They offer little additional information about the relationship between social security and saving. This is partly because the

* I would like to thank Marc Fleurbaey and other participants in the IEA Conference for useful comments.

actual pension system may not be regarded as pure pay-as-you-go financing, especially in Japan.

It is true that pay-as-you-go and fully funded systems are two representative pension schemes, and hence it is useful to investigate and compare the long-run implications of both systems in terms of intergenerational equity. However, the actual public pension system is not exactly the pay-as-you-go system in some countries. In particular, an important feature of Japan's system is that Japan's government has used the general fiscal account to subsidize the social security account. (For example, one-third of the basic pension benefits are subsidized from the general fiscal account; see Takayama, 1998.) One reason why Japan's government has a huge deficit is that its subsidies to the social security account are large. There is no direct link between contributions and benefits in the pension account, unlike the standard unfunded system. Such a subsidy may be regarded as an intergenerational transfer, as it allows the elderly generation to spend more than they contributed at the younger age and it is always easy to get political support for such a subsidy.

In the analysis of the long-run pension system the formulation of two relevant variables, social security benefits and contributions, would crucially affect the result. Actually, the benefit may well be regarded as exogenous and fixed for a long time as the defined benefit system. For example, in Japan the replacement ratio has been maintained at about 65 per cent since the mid-1970s. On the contrary, it seems more plausible to assume that contributions are endogenous. The Japanese government has adjusted old-age benefits and contributions based on the newest estimation of future population changes. There is little room to revise (or reduce) the replacement ratio due to the political pressure of existing old-age generations.

In this chapter we assume that the old-age benefit in real terms, which is represented by the replacement ratio, is a policy variable and hence exogenously given, while social security contributions are effectively determined by interest groups. Thus, the main political issue is to what degree the contributions should be raised.² For example, in Japan labour unions and firms' managers are reluctant to accept an increase in the rate of contributions. They exert political pressures to avoid a large increases in the contribution rate. Many self-employed people are simply not contributing to the basic pension. The contribution avoidance behaviour is illegal, but the pension authority does not effectively penalize it. Still in reality the pension contribution increases much even if they can easily avoid pension contributions under such poor enforcement circumstances.

It seems that all relevant interest groups (such as employees in the private sector, civil servants in the central government, local government employees, the self-employed, firms' managers and so on) may agree with an increase in contributions. Namely, when facing fiscal crises of public sector (or a cut in public goods), every interest group generally agrees with an overall

increase in contributions since it would alleviate the fiscal crisis by reducing subsidies from the government's general account. But this does not necessarily imply that each interest group is willing to accept increases in its own contributions or cuts in its own privilege within the pension system. They would not readily agree with the allocation of increases in total contributions. This phenomenon may be analyzed using a concept of non-cooperative Nash equilibrium.

This chapter offers a theoretical examination of both intragenerational and intergenerational conflicts under the pay-as-you-go system with fiscal subsidies which are provided by the central government. By incorporating 'voluntary' contributions of interest groups to social security at Nash conjectures, our model can exhibit dynamic properties of pension contribution and capital accumulation. The larger the concern for public spending, the deeper accumulation of capital and larger pension contribution would likely occur. Capital accumulation and pension contributions usually grow at the same time. An increase in benefits (or an increase in the replacement ratio) will reduce the level of capital accumulation. However, a good rate of return of providing the contribution will not necessarily lead to large contributions at the second best solution. The pension contribution is too little in terms of the static efficiency (or compared with private consumption) but may be too much or too little in terms of the dynamic efficiency (or as the steady state level). Consumption taxes or combination of a subsidy to social security contributions and an interest income tax would always be desirable. If the government can control the replacement ratio, it would attain the dynamic efficiency by realizing the modified golden rule. The larger the concern for future generations, the more desirable it is to reduce the replacement ratio.

Section 1.2 presents the analytical framework of overlapping generations. Section 1.3 investigates dynamic properties of the model using a Cobb-Douglas example. Section 1.4 considers long-run implications of changes in some policy variables. Section 1.5 examines some normative aspects of public pension policy by deriving optimal taxes on consumption or subsidies on pension contributions. Finally section 1.6 offers some conclusions.

1.2 Model

Analytical framework

We develop a standard model of two-period overlapping generations. An agent (or an interest group) i of generation t born at time t , considers him/herself young in period t , old in period $t + 1$, and dies at time $t + 2$. When young an agent of generation t supplies one unit of labour inelastically and receives wages w_t out of which the agent consumes c_{it}^1 , provides social security contributions g_{it} , pays wage taxes at the rate τ , and saves s_{it} . An agent receives capital income $(1+r_{t+1})s_{it}$ and pension benefits βw_{t+1} when

retired, which the agent then spends entirely on consumption c_{it+1}^2 . There are no private bequests. r_t is the rate of interest in period t . There is no population growth and each generation has n identical individuals (or interest groups). Hence, the population growth rate is always less than the real rate of return on private savings, and the pay-as-you-go system provides a lower rate of return than the funded system, which is relevant for Japan's ageing situation. Thus, younger generations would not like to pay pension contributions unless it provides some additional merits apart from pension benefits.

A member i of generation t faces the following budget constraints:

$$c_{it}^1 = w_{it} - g_{it} - s_{it} - \tau w_{it} \quad (1.1)$$

$$c_{it+1}^2 = (1 + r_{t+1})s_{it} + \beta w_{t+1} \quad (1.2)$$

where βw is old age benefits. β is the replacement ratio, old age benefits per average current wage income. The target contribution level is set by the government, but each interest group can reduce its own contribution by conducting political activities, as explained in section 1.1. Thus, g_i may be regarded as voluntary provision of social security contributions.

His/her lifetime utility function is written as

$$U_t^i = U^i(c_{it}^1, c_{it+1}^2, G_t) \quad (1.3)$$

where G is benefits from public spending, which is a pure public good and beneficial only for the younger aged.

The budget constraint of pay-as-you-go social security account is written as

$$n\beta w_t = z_t + \sum_{i=1}^n g_{it} \quad (1.4)$$

where z measures the subsidy from the government general account. The government budget constraint of this account is given by

$$qG_t + z_t = n\tau w_t \quad (1.5)$$

where q is the marginal (and average) cost of providing the public good. Equations (1.4) and (1.5) summarize the budget constraints including the social security account with fiscal subsidies from the general government account. Due to the subsidy mechanism, there is no direct link between the contributions of the young and old-age benefits in the public pension scheme, in contrast to the standard pay-as-you-go system. A subsidy from the general account of the central government makes the public pension budget constraint 'soft'. In Japan national tax revenues are used for providing basic pension benefits to the current generation.

Although there is no link between contributions and benefits in the above system, each interest group has an incentive to pay contributions since they receive the benefits from public spending, G . By introducing the subsidy mechanism and the benefit of public goods into the pay-as-you-go system, we may explain why the younger generation would like to contribute to public pensions. This is a realistic compromise for the government to collect pension contributions under the situation where it is difficult to penalize the avoidance behaviour of pension contributions. In Japan the government cannot easily collect pension contributions without agreement from the interest groups, as explained in section 1.1. For example, self-employed people could easily avoid pension contributions to the basic pension. Ihori and Itaya (2002, 2003) investigate a similar non-cooperative behaviour of interest groups using the infinite-horizon dynamic game where 'voluntary' contributions alleviate fiscal deficits in the fiscal reconstruction model.

Substituting (1.5) into (1.4), we may derive the overall budget constraint of the public sector as

$$qG_t + n\beta w_t = \tau n w_t + \sum_{i=1}^n g_{it} \quad (1.6)$$

which implies that an increase in pension contributions would result in an increase in public spending so long as the tax and replacement rates are fixed. In this sense, q may be regarded as an index of the (private) rate of return on pension contributions. If q is low, the contribution is very efficient in providing the public good.

From (1.1), (1.2) and (1.6), the lifetime private budget constraint is given by

$$c_{it}^1 + \frac{1}{1+r_{t+1}} c_{it+1}^2 + qG_t = w_t + \frac{1}{1+r_{t+1}} \beta w_{t+1} - n\beta w_t + (n-1)\tau w_t + \sum_{i \neq j} g_{it} \quad (1.7)$$

As in the standard model of voluntary provision of a pure public good, we will exclude binding contracts or cooperative behaviour between the agents and will explore the outcome of non-cooperative Nash behaviour.³

In this Cournot-Nash model, the right-hand side of (1.7) means 'real' income, $E_t^i \equiv w_t(1 - n\beta + (n-1)\tau) + \frac{1}{1+r_{t+1}} \beta w_{t+1} + \sum_{i \neq j} g_{it}$, which contains actual disposable income including current old age benefits, $w_t + \frac{1}{1+r_{t+1}} \beta w_{t+1}$, plus the externalities from other agents' provision of pension contributions and taxes, $\sum_{i \neq j} g_{it} + (n-1)\tau w_t$ minus total old age benefits, $n\beta w_t$.

From (1.6) and (1.7) we have

$$\sum_{i=1}^n E_t^i = n(1 - \beta)w_t + \frac{1}{1+r_{t+1}} n\beta w_{t+1} + q(n-1)G_t \quad (1.8)$$

Namely, add (1.7) from $i=1$ to n and use (1.6). Then we will get (1.8). $q(n-1)G_t$, the third term in the right-hand side of (1.8), captures externalities from $n-1$ other persons' contributions within the same generation.

Let us then formulate the aggregate production function. The firms are perfectly competitive profit maximizers who produce output using the production function.

$$Y_t = F(K_t, n_t) = K_t^{1-\lambda} n_t^\lambda \quad (0 < \lambda < 1) \quad (1.9)$$

As for the standard first-order conditions from the firm's maximization problem in period t , we have

$$r_t = r(K_t) \quad (1.10)$$

$$w_t = w(K_t) \quad (1.11)$$

since n is exogenously given. Since we follow the standard Diamond-type overlapping generations growth model with productive capital, capital does not depreciate at all.

In an equilibrium agents can save by holding physical capital. We have

$$ns_t = K_{t+1} \quad (1.12)$$

The system may be summarized by these two equations.

$$\begin{aligned} nE[Q(K_{t+1}), q, U_t] &= n(1-\beta)w(K_t) + Q(K_{t+1})n\beta w(K_{t+1}) \\ &\quad + q(n-1)E_2[Q(K_t), q, U_t] \end{aligned} \quad (1.13)$$

$$nE_1[Q(K_{t+1}), q, U_t] = K_{t+1}/Q(K_{t+1}) + n\beta w(K_{t+1}) \quad (1.14)$$

where $E(\cdot)$ is the expenditure function, which minimizes the left-hand side of (1.7) as a function of $Q(K) \equiv 1/(1+r(K))$, q , and U . $E_2 \equiv \partial E/\partial q = G$ is the compensated demand function for G and $E_1 \equiv \partial E/\partial Q = c^2$ is the compensated demand function for c^2 . It is assumed that each individual's expectation on the rate of interest for his old age is perfect foresight. This assumption is made only for the sake of simplicity. If we assume the static expectation, the analytical results would qualitatively be the same because we focus on the steady state properties of the system.

It is well known that in the voluntary provision of public good model redistribution of income does not matter at the interior solution (see, among others Warr, 1983). This neutrality result also holds in the present model. However, since our main concern is with intergenerational equity and the efficiency of the pension system, we do not investigate the issue of intragenerational redistribution by assuming that all interesting groups are identical.

We have assumed identical agents for analytical simplicity. If we allow for heterogeneity, the analytical framework would become complicated, but the main qualitative results below would hold as in the present model.

1.3 Cobb-Douglas example

In order to demonstrate concrete results with respect to dynamic properties, let us assume that the utility function (1.3) is given as a Cobb-Douglas one. The qualitative results are almost the same as in a more general production function.

$$U_t = (c_t^1)^{\alpha_1} (c_{t+1}^2)^{\alpha_2} (G_t)^{\alpha_3} \quad (\alpha_1 + \alpha_2 + \alpha_3 = 1) \quad (1.3)'$$

Then in this case we have

$$c_t^1 = \alpha_1 E_t \quad (1.15-1)$$

$$c_{t+1}^2 = \alpha_2 (1 + r_{t+1}) E_t \quad (1.15-2)$$

$$G_t = \alpha_3 E_t / q \quad (1.15-3)$$

From (1.2) (1.12) and (1.15), (1.14) may reduce to

$$nq\alpha G_t = K_{t+1} + \frac{1}{1 + r(K_{t+1})} n\beta w(K_{t+1}) \quad (1.16)$$

where $\alpha \equiv \alpha_2 / \alpha_3$.

From (1.16) we have

$$G_t = G(K_{t+1}) \quad (1.17)$$

where

$$G'(K) = \frac{1}{nq\alpha} \{1 + A(K)\} > 0 \quad (1.18)$$

and

$$A(K) \equiv \frac{\beta\lambda(1 - \lambda)n^\lambda K^{-\lambda}(1 + n^\lambda K^{-\lambda})}{(1 + (1 - \lambda)n^\lambda K^{-\lambda})^2}.$$

From (1.18) we know $G'(\infty) = \frac{1}{nq\alpha}$ and $G'(0) = \frac{1+\beta\lambda}{nq\alpha}$. It is also easy to show $G'' < 0$.

G and K always move in the same direction, which implies a positive relation between G and K . Under the interdependence between the pension benefits and public spending, formulated as (1.4) and (1.5), an increase in public spending (or pension contribution) is consistent with economic growth. An increase in capital stock raises real income, stimulating the demand for public goods. In order to have a larger amount of public goods,

it is necessary to reduce the subsidy from the general account to the pension account. Thus, the agent is willing to pay more pension contributions, making larger public spending possible. In general we have the substitution effect as well. Namely, an increase in K raises $1/(1+r)$, the intertemporal price of c^2 , inducing a substitution from c^2 to G . This effect also produces larger pension contributions.

Hence, considering (1.10)(1.11)(1.12-1) and (1.12-3), (1.13) may be rewritten as

$$\frac{qn}{\alpha_3}G(K_{t+1}) = n(1-\beta)w(K_t) + \frac{n\beta}{[1+r(K_{t+1})]}w(K_{t+1}) + q(n-1)G(K_{t+1}) \quad (1.19)$$

Now we have

$$q(\theta n + 1 - n\alpha)G(K_{t+1}) + K_{t+1} = n(1-\beta)w(K_t) \quad (1.20)$$

where $\theta = (1 - \alpha_3)/\alpha_3$. (1.20) may be expressed as

$$K_{t+1} = \Phi(K_t) \quad (1.20')$$

which is the fundamental dynamic equation of the model.

Let us now investigate dynamics of (1.20) or (1.20)'. We have from (1.20)'

$$\Phi' = \frac{(1-\beta)q^{-1}n^\lambda K^{-\lambda}(1-\lambda)}{(\theta n + 1 - n\alpha)G' - q^{-1}} \quad (1.21)$$

The numerator when $K \Rightarrow \infty$

$$(\theta n + 1 - n\alpha)G' - \frac{1}{q} = \frac{1}{nq\alpha}(\theta n + 1)$$

is positive since $\theta > \alpha$ and hence $\theta n + 1 > n\alpha$. Thus, (1.21) is positive. We know that $\Phi'(0) > 1$.

The stability condition is

$$\Phi' = \frac{(1-\beta)q^{-1}n^\lambda K^{-\lambda}(1-\lambda)}{(\theta n + 1 - n\alpha)G' - q^{-1}} < 1$$

Or, we have

$$\Delta \equiv (\theta n - n\alpha + 1)G' - q^{-1} - (1-\beta)q^{-1}n^\lambda K^{-\lambda}(1-\lambda) > 0 \quad (1.22)$$

We assume

$$\theta n - 2\alpha n + 1 > 0$$

Then, we have that $\Phi'(\infty) < 1$. The system becomes dynamically stable. The larger the level of α_3 (the preference for public spending), it is more likely that the system would be stable.

1.4 Comparative statics

From (1.13) and (1.14) the steady-state equilibrium may be summarized by the following two equations.

$$nE[Q(K), q, U] = n[1 - \beta + \beta Q(K)]w(K) + q(n-1)E_2[Q(K), q, U] \quad (1.23)$$

$$nE_1[Q(K), q, U] = \frac{1}{Q(K)}K + n\beta w(K) \quad (1.24)$$

Totally differentiating (1.23) and (1.24), we have

$$\begin{aligned} \frac{dU}{d\beta} = \frac{1}{\Omega} \left\{ nw(-1+Q) \left[nE_{11}Q' - \beta w' - \frac{Q-Q'K}{Q^2} \right] \right. \\ \left. - [(nE_1 - q(n-1)E_{21})Q' - n(1-\beta+\beta Q)w' - n\beta wQ']nw \right\} \quad (1.25-1) \end{aligned}$$

$$\frac{dK}{d\beta} = \frac{nw}{\Omega} [nE_U - q(n-1)G_U - (-1+\theta)nE_{1U}] < 0 \quad (1.25-2)$$

$$\begin{aligned} \frac{dU}{dq} = \frac{1}{\Omega} \left\{ [bE_2 + q(n-1)E_{22}] \left[nE_{11}Q' - \beta w' - \frac{Q-Q'K}{Q^2} \right] \right. \\ \left. - (-nE_{12})[(nE_1 - q(n-1)E_{21})Q' - n(1-\beta+\beta Q)w' - n\beta wQ'] \right\} \quad (1.25-3) \end{aligned}$$

$$\frac{dK}{dq} = \frac{n}{\Omega} [-E_{12}[nE_U - q(n-1)G_U] - E_{1U}(n-1)(E_2 + qE_{22})] \quad (1.25-4)$$

where

$$\begin{aligned} \Omega \equiv [nE_U - q(n-1)G_U] \left[nE_{11}Q' - \beta w' - \frac{Q-KQ'}{Q^2} \right] \\ - nE_{1U}[(nE_1 - q(n-1)E_{21})Q' - n(1-\beta+\beta Q)w' - n\beta wQ'] \end{aligned}$$

We know $nE_U - q(n-1)E_{2U} > 0$, $E_{1U} > 0$. $nE_{11}Q' - \beta w' - \frac{Q-KQ'}{Q^2}$ is negative. And the sign of $(nE_1 - q(n-1)E_{21})Q' - n(1-\beta+\beta Q)w' - n\beta wQ'$ is ambiguous. When Ω becomes negative, an increase in exogenous resources will be beneficial, which is intuitively plausible. Thus, we assume $\Omega < 0$.

The effect of an increase in β or q on U is generally ambiguous. By setting (1.25-1) to be zero, we may implicitly derive the optimal level of β , the replacement ratio at the second best solution. An increase in β raises old-age consumption, which is desirable, while it reduces the effective income, the

right-hand side of (1.23), which is undesirable. It is true that an increase in β raises old-age welfare, but it reduces young-age welfare. The overall welfare effect is ambiguous, which is intuitively plausible.

The sign of (1.25-3) is also ambiguous. It should thus be stressed that an increase in q could raise welfare at the second best solution. Intuition of this paradoxical result is as follows. On the one hand, an increase in q raises private savings due to the substitution effect, which may be beneficial. On the other hand, it reduces the (private) rate of return on contributions by raising the marginal cost of producing public goods, which income effect is not beneficial. The overall effect is thus ambiguous.

We know the sign of (1.25-2) becomes negative. As to the effect on capital accumulation, an increase in the replacement ratio will reduce accumulation of capital. The sign of (1.25-4) is ambiguous. On the one hand, an increase in q reduces the lifetime disposable income, producing the negative income effect on capital accumulation. On the other hand, an increase in q will reduce voluntary contributions, producing more savings due to the substitution effect. This stimulates capital accumulation. Thus, the overall effect on capital accumulation is ambiguous. In other words, an increase in the (private) rate of return of providing the contribution (a decreasing in q) may not necessarily stimulate accumulation of capital if the substitution effect is large.

The comparative static results suggest that there may exist a conflict between the first best and second best situations when q changes; a decrease in q is desirable at the first best solution but is not always desirable at the second best solution. The public sector does not have a strong incentive to raise the (private) rate of return of providing contributions when more capital accumulation is needed.

1.5 Normative aspects of public pension contributions

First best solution

In order to investigate the normative aspect of the model, it is useful to derive the first best solution. From (1.1)(1.2)(1.4)(1.5) and (1.9), the feasibility condition is given as

$$Y_t + K_t = K_{t+1} + n(c_t^1 + c_t^2) + qG_t \quad (1.26)$$

We analyze the optimal growth path which would be chosen by a central planner who maximizes an intertemporal social welfare function expressed as the sum of generational utilities discounted by the social discount factor on future generations, ρ , which is between 0 and 1. The discounted social objective is standard in the optimal growth literature, so I follow this. But it

may be problematic because later generations are penalized.

$$\text{Max} \sum_{t=0}^{\infty} \rho^t U(c_t^1, c_{t+1}^2, G_t) \text{ subject to} \quad (1.26)$$

In other words, the first best problem is to maximize the Lagrange function

$$W = \sum_{t=0}^{\infty} \rho^t \{nU(c_t^1, c_{t+1}^2, G_t) - \mu_t [qG_t - Y_t - K_t + K_{t+1} + n(c_t^1 + c_t^2)]\} \quad (1.27)$$

where $\rho^t \mu_t$ is a Lagrange multiplier at time t .

The first-order conditions are as follows.

$$U_{1t} - \mu_t = 0 \quad (1.28-1)$$

$$U_{2t+1} - \mu_{t+1} \rho = 0 \quad (1.28-2)$$

$$nU_{3t} - q\mu_t = 0 \quad (1.28-3)$$

$$\mu_{t+1}(1 + r_{t+1})\rho - \mu_t = 0 \quad (1.28-4)$$

along with the transversality conditions

$$\lim_{t \rightarrow \infty} \rho^t \mu_t G_t = 0, \quad \lim_{t \rightarrow \infty} \rho^t \mu_t K_t = 0$$

where $U_{1t} = \partial U_t / \partial c_t^1$, $U_{2t+1} = \partial U_t / \partial c_{t+1}^2$, and $U_{3t} = \partial U_t / \partial G_t$.

From these conditions we have

$$\frac{U_{3t}}{U_{1t}} = \frac{q}{n} \quad (1.29-1)$$

$$\frac{U_{3t}}{U_{2t+1}} = \frac{q(r_{t+1} + 1)}{n} \quad (1.29-2)$$

From (1.29-1) and (1.29-2), we have

$$\frac{U_{3t}}{U_{1t}} + \frac{U_{3t}}{U_{2t+1}} < q + q(1 + r_{t+1}) \quad (1.30)$$

Note that $n > 1$. Since in the competitive economy we always have

$$\frac{U_{3t}}{U_{1t}} = q, \quad (1.31-1)$$

$$\frac{U_{3t}}{U_{2t+1}} = q(1 + r_{t+1}), \quad (1.31-2)$$

inequality (1.30) means that the public good-private consumption ratio $G_t/(c_t^1 + c_{t+1}^2)$ in the competitive economy is smaller than in the first best economy. Each agent does not fully recognize the total effect of voluntary contributions to public spending. That is, the main reason for such underprovision of G in the relative sense is that each group disregards a positive externality of cooperation with public pension contribution which spills over into all other groups in choosing its own contribution [This is the conventional result in the literature. See Bliss and Nalebuff (1984), Bergstrom et al. (1986), Boadway et al. (1989) and Ihuri and Itaya (2002)(2003).] The pension contribution (or public spending) divided by private consumption is too little in the pension system with public subsidies at the second best solution. In this sense, the pension contribution is too little and private consumption is too much in the competitive economy in terms of the static efficiency.

In terms of the dynamic efficiency from (1.28-4) we have as the modified golden rule:

$$(1+r)\rho = 1 \quad (1.32)$$

The first best solution may be summarized by (1.26), (1.29-1), (1.29-2) and (1.32).

Under the Cobb-Douglas utility function (1.3)', (1.29-1) and (1.29-2) are rewritten as

$$\frac{\alpha_3 c^1}{\alpha_1 G} = \frac{q(r+1)}{(1+r)n} \quad (1.29-1)'$$

$$\frac{\alpha_3 c^2}{\alpha_2 G} = \frac{q(r+1)}{n} \quad (1.29-2)'$$

Substituting these two equations into (1.26) and considering (1.32), we obtain the first best G , G_{FB} , as

$$G_{FB} = \frac{Y_{FB}}{\frac{q}{\alpha_3} \frac{1}{\rho} (\rho\alpha_1 + \alpha_2) + q} \quad (1.33)$$

where Y_{FB} is output associated with the modified golden rule. The first best pension contribution is decreasing with q and increasing with ρ and α_3 . These results are intuitively plausible. For example, the larger the concern for future generations (ρ), the larger the first best levels of capital accumulation and public goods.

Steady-state level of pension contribution

In the standard overlapping generations growth model it is well known that capital may be over-supplied in the competitive equilibrium. Capital may be

too much in this model as well when the competitive steady-state economy is on the inefficient path ($(1+r)\rho < 1$), and vice versa. The smaller the concern for the future generations, it is more likely to have the inefficient case.

Let us compare the steady-state levels of pension contribution or public spending, G^* at the Nash solution given by (1.23)(1.24), and G_{FB} at the first best solution given by (1.33). Remember that the pension contribution at the second best solution is independent of ρ , while G_{FB} is increasing with ρ . Hence, the steady-state level of pension contribution may be too much if q is very high or ρ is very small. If $r^* \geq \frac{1-\rho}{\rho}$, it is easy to see $G^* < G_{FB}$. However, if $r^* < \frac{1-\rho}{\rho}$, we cannot exclude the possibility of $G^* > G_{FB}$. Pension contributions at the non-cooperative Nash solution could be over-provided. In other words, G^* given by (1.23)(1.24) could be higher than G_{FB} given by (1.33) if ρ is small enough. If α_2 and q are large, we could have the same possibility. The pension contribution at the second best solution may be increasing with q , while its steady-state level at the first best solution is decreasing with q . Thus, when q is high, it is likely that the pension contribution is too much even if each interest group can easily avoid contributions.

In Figure 1.1 line AB represents the feasibility condition at the steady state where the modified golden rule (1.32) is satisfied.

$$n(c^1 + c^2) - bG = Y_{FB} \quad (1.34)$$

Point F is the first best solution, while point N is the Nash equilibrium when $r^* = \frac{1-\rho}{\rho}$. The movement from F to N on the same budget line AB reflects the free-riding effect. As shown in Figure 1.1, G is too little at point N compared with point F.

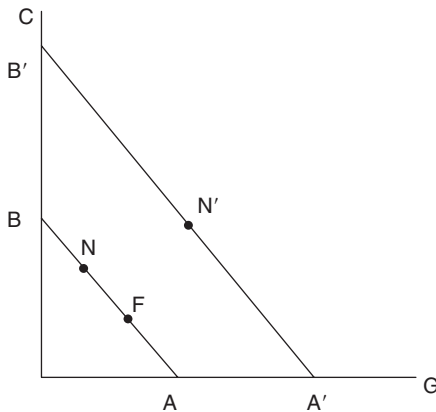


Figure 1.1 Steady-state level of pension contributions

If $r^* < \frac{1-\rho}{\rho}$, we may draw Figure 1.1 where line A'B' represents the feasibility condition at the steady state in the competitive economy. A shift from line AB to line A'B' reflects the income effect. Thus, if the income effect dominates the free-riding effect, G^* at point N' could be greater than G_{FB} at point F. The lower the discount factor ρ and the concern with future consumption α_2 , it is more likely to have such a paradoxical case. In such a case the pension contribution is too much even if each interest group can easily avoid contributions and the government cannot effectively penalize the avoiding behaviour.

If we allow for positive population growth, then the modified golden rule (1.32) will be altered to

$$(1+r)\rho = 1 + \gamma \quad (1.32)'$$

where γ is the population growth rate. In this case the competitive steady-state economy is on the efficient path if and only if $(1+r)\rho > 1 + \gamma$. Hence, even if ρ is close to 1, it is still possible that the path is inefficient where $\gamma > r$, and we cannot exclude the possibility of $G^* > G_{FB}$.

Optimal tax and subsidy policy

Finally, let us consider some tax and subsidy policy to attain the first best economy in the long run. Suppose that consumption taxes η^1, η^2 , a subsidy to contributions ε and a tax on interest income μ are available. Then, the consumer's budget constraints (1.1) and (1.2) are rewritten as

$$(1 + \eta^1)c_{it}^1 = (1 - \tau)w_{it} - (1 - \varepsilon)g_{it} - s_{it} \quad (1.1)'$$

$$(1 + \eta^2)c_{it+1}^2 = [1 + (1 - \mu)r_{t+1}]s_{it} + \beta w_{t+1} \quad (1.2)'$$

Then, considering (1.1)' and (1.2)', the first-order conditions (1.31-1) and (1.31-2) are rewritten as

$$\frac{U_{3t}}{U_{1t}} = \frac{q(1 - \varepsilon)}{1 + \eta^1}, \quad (1.3)'$$

$$\frac{U_{3t}}{U_{2t+1}} = \frac{q[1 + (1 - \mu)r_{t+1}]}{1 + \eta^2}, \quad (1.4)'$$

From (1.29-1)(1.29-2) and (1.31-1)'(1.31-2)', the optimal conditions are given as

$$\frac{q(1 - \varepsilon)}{1 + \eta^1} = \frac{q}{n}$$

$$\frac{q[1 + (1 - \mu)r]}{1 + \eta^2} = \frac{q(1 + r)}{n}$$

Hence, the optimal values of $\varepsilon, \mu, \eta^1, \eta^2$ are not uniquely determined. For example, when $\eta^1 = \eta^2 = 0$, the optimal values of ε and μ are respectively given as

$$\varepsilon^* = 1 - \frac{1}{n} > 0 \quad (1.35-1)$$

$$\mu^* = \frac{(1+r)(n-1)}{rn} \quad (1.35-2)$$

(1.35-1) means that a subsidy to contributions is used for attaining the static efficiency. In Japan public pension contributions are fully exempted from the income tax base. It actually subsidizes contributions. Such a subsidy to contributions may be justified to realize the static efficiency although the actual level of subsidies may not be optimal. From (1.35-2) it is desirable to tax interest income to stimulate pension contributions. Note that a tax on old-age benefits is not effective since it cannot affect the first order conditions of consumers.

Alternatively, when $\varepsilon = \mu = 0$, the optimal values of consumption taxes are respectively given as,

$$\eta^{1*} = \eta^{2*} = n - 1 > 0 \quad (1.36)$$

The optimal consumption tax rates are uniform and positive. Intuition is as follows. By taxing private consumption, providing pension contributions so as to supply public goods becomes more favourable. It would thus stimulate pension contributions, which is desirable to internalize the free-riding behaviour of interest groups.

As to the dynamic efficiency, intergenerational redistribution policy would be useful. Or if the government can control the replacement ratio, it would attain the dynamic efficiency by realizing the modified golden rule. From (1.25-2) the replacement ratio affects capital accumulation negatively. So the government may choose the replacement ratio to attain (1.32). When physical capital is over-provided at the Nash equilibrium ($r^* \leq \frac{1-\rho}{\rho}$), the intergenerational transfer from the young to the old would be desirable. An increase in the replacement ratio could have this effect. Such a policy would reduce the lifetime disposable income, reducing savings.

On the contrary, when capital is under-provided ($r^* > \frac{1-\rho}{\rho}$), the intergenerational transfer from the old to the young such as a decrease in the replacement ratio would be desirable. In this case the pension contribution is too little due to the under-accumulation of capital, and it is thus necessary to stimulate savings. The larger the concern for future generations, it is more likely to have this case.

1.6 Conclusion

Suppose the pay-as-you-go system has to be maintained as the means of intergenerational transfer. Without effective enforcement measures to collect pension contributions from various interest groups, the government would face the difficulty of maintaining the pay-as-you-go system in an ageing society. This chapter has shown that interest groups 'voluntarily' provide a social security contribution if a part of pension benefits is financed by subsidies from the general account of national government. This is so because, under this subsidy mechanism each interest group has an incentive to contribute to the pay-as-you-go pension system even if the population growth rate is less than the real rate of interest.

An increase in capital stock raises real income, stimulating the demand for public goods. In order to have a larger amount of public goods, it is necessary to raise pension contributions. Thus, the agent is willing to accept more pension contributions, resulting in larger public spending. The larger pension contribution and capital accumulation would be likely to coexist at the competitive solution. In this sense, the subsidy mechanism is a realistic compromise to let each interest group cooperate with the otherwise unpopular pay-as-you-go system in the situation of an ageing population.

We have also clarified how the relevant parameters would affect dynamic properties. An increase in the replacement ratio will reduce accumulation of capital, although its effect on welfare is ambiguous. There may exist a conflict between the first best and decentralized situations when the cost of public goods changes. The public sector does not have a strong incentive to raise the (private) productivity of providing pension contribution although it benefits all generations at the first best solution.

It is well known that capital may be too much in the long run when the competitive steady-state economy is on the inefficient path. Even if the concern for future generations are large, it is still possible to have such an inefficient case. We have shown that in such a case, the steady-state level of pension contributions may be too much in the long run, although it is too little compared with consumption. The pension contribution could be too much even if each interest group can easily avoid contributions and the government cannot penalize the avoiding behaviour.

As to attaining the static efficiency, consumption taxes or combination of a subsidy to social security contributions and an interest income tax would be useful for correcting the free-riding behaviour of interest groups. This is desirable even if capital is over-accumulated. If the government can control the replacement ratio, it can attain the dynamic efficiency by realizing the modified golden rule. When capital is under-provided, the intergenerational transfer from the old to the young such as a decrease in the replacement ratio would be desirable. The larger the concern for future generations, the more likely it is to have this case.

Notes

1. Abel (1999) develops a tractable stochastic overlapping generations model to analyze the equilibrium equity premium and growth rate of the capital stock in the presence of a defined-benefit social security system.
2. From the 2004 pension reform Japan is moving from the defined benefit system to the defined contribution system. See Takayama (2007).
3. As for voluntary provision of public goods, see Shibata (1971), Warr (1983), and Bergstrom et al. (1986) among others.

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