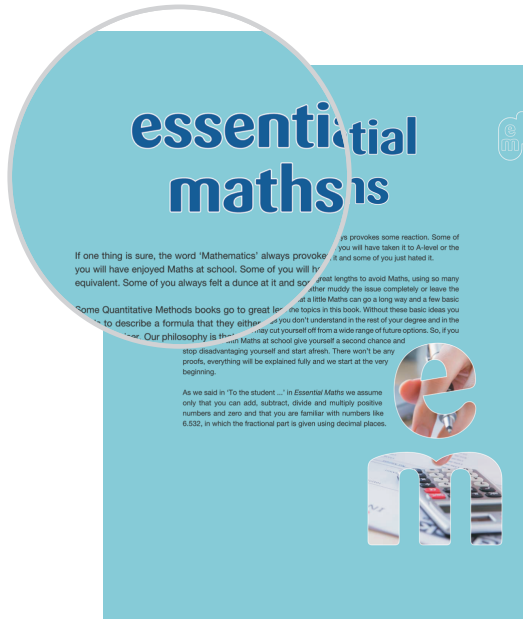


# Guided tour of the book



✉ **Section summary** An overview of each main section

✉ **Contexts** Places each chapter in context and tells what you need to know to get the most from it

✉ **Objectives** What you should be able to do after completing the chapter



**Icons** Pointing out tips, warnings and where computers can help

**Key facts** Summaries of essential results and equations

Numbers and symbols 25

To solve problems and solve equations we need to know how to manipulate symbols. This is a problem if we remember that the symbols merely stand in for the numbers and so are used in exactly the same way. In this section we recall the work of Sections 1–3, but particularly to expressions containing symbols.

**Adding and subtracting**  
 SAME SIGNS  
 $-(-a)$  or  $+(+a)$  gives  $+a$   
 $-(+a)$  or  $-(-a)$  gives  $-a$   
 or expression.

**Multiplying and dividing**  
 SAME SIGNS  
 multiplying  $a \times b = ab$   
 $(-a)(-b) = ab$   
 dividing  $a \div b = \frac{a}{b}$   
 and  $(-a) \div b = -\frac{a}{b}$   
 dividing  $(-a) \div b = -\frac{a}{b}$   
 $a \div (-b) = -\frac{a}{b}$

**check these**

$2 + (-a) = 2 - a$   
 $3 - (-a) = 3 + a$   
 $2 \times (-a) = -2a$   
 $(-3) \times (-a) = 3a$   
 $(-b) \times c = -bc$

**Icons** For easy access

**Test boxes** A chance to see whether you already know the material in this section. If you do, you can skip it!

**Check this** Worked examples so that you can learn by doing

60 Essential maths 2

**6 Factorising**

**test box 6**  
 $y^2 + 4y - 5, \quad a^2 + c^2 + 2ac$   
 $(y + 5)(y - 1), \quad (a + c)^2$

**Factorise**  $y^2 + 4y - 5$  and  $a^2 + c^2 + 2ac$

**Solution** When expanding the brackets may help to simplify an expression, it will be useful when we come to solving equations in Chapter 6M3, is by

**Factorising?**  
 Factorising is the opposite of expanding brackets. To expand an expression we take it and write down its equivalent without the brackets, whereas to factorise, we take an expression and we insert some brackets. At the moment this may seem a rather futile operation, but we will see that it often enables us to cancel terms.

**Factorising**  
 We take an expression – or part of it – and replace it with two or more factors, which multiply together to give the original expression. We are expressing a product. For example,  $3x + 6$  factorises to  $3(x + 2)$ . Your solution is correct by multiplying out the brackets again; that is,  $3(x + 2) = 3x + 6$  should always get back to the original expression.

**check these**

**Decide which** factorisations, a or b, are correct:

$4 + 8y$  factorises to a.  $4(1 + 2y)$  or b.  $4(1 + y)$   
 $3x + 6$  factorises to a.  $3x(x + 2)$  or b.  $y(3x + 2)$

$3xy + 6$  factorises to a.  $3y(x - 1)$  or b.  $3y(x + 1)$

$ab + b - 2a - 2$  factorises to a.  $(a + 1)(b - 1)$  or b.  $(a + 1)(b - 2)$

**Solution:**  
 Multiply out the brackets for each a. and b. to see which gives the original expression. For instance, in the second example a. multiplies out to  $3yx + 6y$  which is clearly not equal to  $3xy + 6$ . The solutions are a., b., a., a., b.

**Why factorise?**  
 To demonstrate the benefits of factorising let's take the expression

$$\frac{pq^2 - p}{q^2 - 1}$$

At present, it looks rather complicated, so it would be nice to simplify it. The numerator should look familiar to you as we factorised it as  $p(q - p) = p(q - 1)$  in the Check these above. When we replace  $pq^2 - p$  in the original expression with the factorisation we obtain

of the values are between  $20 - (2 \times 5)$  and  $20 + (2 \times 5)$ . That is, at least three-quarters of the items in the sample lie between 10 and 30. In the same way, Chebyshev's result for  $k = 3$  tells us that at least

$$1 - \frac{1}{3^2} = 88.9\%$$

of the data lies between  $20 - (3 \times 5)$  and  $20 + (3 \times 5)$ , so between 5 and 35 and the result for  $k = 4$  tells us that at least

$$1 - \frac{1}{4^2} = 56.25\%$$

**Work cards** Exercises with fully worked solutions at the end of each section

**Work card**

1. Use the method given in Section 1.1 to calculate the variance of the following sample. Lay out your working in columns.

45      4      45      50

2. In a survey 500 respondents were asked to state their average income in pounds per year. The results are as follows. Calculate the mean and standard deviation of the data.

- Which of the following statements is true? Explain your answers.
- (i) At least 37% of respondents have a salary that is more than £28,320.
  - (ii) At most 12% of respondents have a salary that lies between £32,700 and £33,330.
  - (iii) Example 1.4.4. So your working should be:

Data	Data - $\bar{x}$	(Data - $\bar{x}$ ) <sup>2</sup>
45	1	1
42	-2	4
38	-6	36
45	1	1
50	6	36
<b>78 so <math>\bar{x} = \frac{78}{4} = 19.5</math></b>		

2. (i) True, take  $k = 2$ . (ii) True from (i). At most, 12% of respondents have a salary that is more than £28,320. (iii) False. It is true that at least 89% (approximately) of respondents have salaries between £32,700 and £33,330.

**Assessments** Additional exercises – without solutions

**Assessment**

1. At a glance say which of these two data sets has the largest variance. Explain your answer.

A      3   5   7   9  
B      3   7   7   7

2. A friend wishes to invest in three ordinary shares. For each company the percentage returns over the last 10 years are given below.

	Mean (%)	Standard (%)
Company A	8	5
Company B	5	2
Company C	5	1

On the basis of this information

- a. Which company would you advise against?
- b. If your friend is 'very well-off' and is really making the investment for 'fun', which would you recommend?
- c. If your friend requires a relatively secure ordinary share investment, which of these companies would you recommend?

3. In its end of year report the Finance and Economics Faculty of a university publishes that 200 students obtained a degree that year. These had an average overall percentage mark of 59%, with a standard deviation of 10%.

On the basis of this information alone make a statement concerning

- a. The number of students who obtained between 39% and 79%.
- b. The number of students who obtained less than 39% or more than 79%.
- c. The number of students who obtained more than 89%.
- d. A range of marks which 50% of students attained.

**3  $\Sigma$  and a short-cut for variance**

There is a quicker method of calculating the variance of a sample. To explain it we need to introduce a special symbol,  $\Sigma$ , called 'sigma', which means, 'the sum of'. Sigma is widely used in statistics as well as in mathematics.

**Introducing the summation sign  $\Sigma$**

In Section 1 we said that we would label a set of data  $x_1, x_2, x_3, \dots, x_n$ . This enables us to write down formulae for functions of the data. For instance, the sum of the first 2 items of the data is

$$x_1 + x_2$$

**Further reading** Pointers to more sources of information about specific topics



**Further reading:**

- Anderson *et al.* (Statistics, chapter on forecasting as does Newbold *et al.*)
- Hillier and Lieberman include chapter on forecasting.
- Mendenhall *et al.*, where forecasting is also called time series.
- Newbold *et al.* includes a clear chapter on autoregressive moving average models.