

# ESSENTIAL MATHS

## EM 1 Numbers and symbols

### ASSESSMENT FOR 1 AND 2

Sections 1 and 2 revise arithmetic using negative numbers. This does not usually cause problems.

- a.  $-20 + 10 + 5 = -5$   
b.  $-20 + 10 - 5 = -15$   
c.  $-1 + 1 - 1 - 1 = -2$
- a.  $-50$  b.  $-12$  c.  $-150$  d.  $-20$  e.  $-20$  f.\*  $-32$   
g.\*  $-128$  h.  $20$  i.  $-5$  j.  $-5$  k.  $5$

\*The students will have to work f and g out cumulatively,  $-2 \times -2 = 4$ ,  
 $-2 \times -2 \times -2 = 4 \times -2 = -8$ , and so on.

### ASSESSMENT 3

Section 3 describes the order of operations. Notice that like most texts we give equal priority to multiplication and division and then work the operations from left to right. Some texts give multiplication priority over division (the BOMDAS rule). In practice brackets are usually used to avoid any ambiguity anyway.

- a.  $20 + 12 = 32$   
b.  $-5 - 6 + 1 = -10$   
c.  $6 \div 6 + 4 = 1 + 4 = 5$   
d.  $3 \times 3 \times 3 + 3 = 27 + 3 = 30$   
e.  $3 \times 3 \times 3 + 3 = 30$  as in d.  
f.  $\frac{9}{3} = 3$   
g.  $\frac{8}{4} \div (-2) = 2 \div (-2) = -1$
- a.  $3 + (50 \div 10 + 1) = 3 + (5 + 1) = 9$   
b.  $\frac{7}{(-36) \times 4} = -\frac{7}{144}$

$$\text{c. } \frac{40}{8} \left( 3 + \frac{2}{2} \times 3 \right) + 5 = 5(6) + 5 = 35$$

$$\text{d. } \frac{20\,000}{(1000 - 500) \times 2} - 2 = \frac{20\,000}{500 \times 2} - 2 = 20 - 2 = 18$$

$$\text{e. } \frac{39}{13} + 5 \times \frac{110}{11} = 3 + 5 \times 10 = 53$$

## ASSESSMENT 4

Using a calculator, rough estimates and rounding. The students will think they know how to use a calculator, but here we are trying to instill an awareness of what they are doing, an intuitive 'feel' for magnitude and the sensible answer, thus the 'rough estimates' section.

- 14.8642; 218.988
- a. 32.90    b. 1.0939    c. \$142

For example, for **a.**, the aircraft does 9000 km at roughly 3 km per litre, so uses approximately 3000 litres which cost about £1 each. So total annual costs are about £4000 which is £33.33 per member per month. The only answer close to this is £32.90.

- 13.6667; -3.1560; 200 000.0000; 157.0000; 55.1235.
- 14;                      -3.2;                      200 000;                      160;                      55
- $3.000\,051 \times 10^6$ ;  $9.142 \times 10^{-7}$ ;  $-1.0201 \times 10^2$ ;  $4.14 \times 10^0$
- 342 000 000; 0.000 001 004; 9990

## ASSESSMENT 5

Now we introduce symbols without frightening too much and by trying to convince the reader that symbols are useful. Later in the degree program Maths is usually encountered in the context of 'Financial modelling' or 'Econometric modelling' and so on and so we introduce the 'model' terminology here, at the same time as equations. It is difficult to teach someone to formulate an equation, particularly because most of us (instructors) do it intuitively. However, in the section headed 'Using symbols to represent relationships' I have tried to break down the problem into reading once to assign the symbols and then reading again and translating the information given into symbols. The students need to use a spreadsheet for Question 4, although this section is the only place in which we use spreadsheets explicitly.

- Let  $w$  = width of second room,  
 $b$  = breadth of second room.

Dimensions of first room are then  $1.5w$  and  $2b$ , so area of first room is

$$1.5w \times 2b = 3wb.$$

Area of second room is  $wb$  so total area of flat is

$$3wb + wb = 4wb.$$

2. Let  $f$  = number of French francs purchased.

a. By credit card, to buy  $f$  francs I will need  $f/8.25$  pounds and I will have to pay 2.3% of this amount in commission, i.e. I will be charged

$$\frac{f}{8.25} + \frac{2.3}{100} \times \frac{f}{8.25} \text{ or } \frac{f}{8.25} \times 1.023$$

in all.

b. From a Bureau de Change I will be charged  $f/8.5$  pounds plus commission of £3 plus 2% of this amount, i.e.

$$\frac{f}{8.5} + \frac{2}{100} \times \frac{f}{8.5} + 3$$

or

$$\frac{f}{8.5} \times 1.02 + 3.$$

Students who have more maths background will be able to simplify the expressions obtained in **a.** and **b.** further.

(i) When  $f = 700$     **a.** becomes 86.8  
                                   **b.** becomes 87

When  $f = 800$     **a.** is 99.2  
                                   **b.** is 99

When  $f = 1000$     **a.** is 124  
                                   **b.** is 123

(ii) Students should evaluate both costs for a range of arbitrarily chosen values of  $f$  and include them in a table with those already evaluated in (i) For instance,

$f$	<b>a.</b> CC	<b>b.</b> B de C
650	80.6	81
700	86.8	87
750	93	93
800	99.2	99
850	105.4	105
1000	124	123

Students who arbitrarily chose 750 will discover that the costs are the same for this number of francs. Other students ought to be able to see that for a low  $f$  the Bureau de Change is more expensive, whereas as  $f$  increases, the difference in cost decreases until (above 750) the Bureau de Change becomes cheaper, and make a guess, somewhere near 750, for the number of francs at which the costs are the same.

3. Aircraft does  $n$  miles a year, so uses  $\frac{n}{9}$  gallons a year. Cost of fuel therefore is

$$3 \times \frac{n}{9}$$

Maintenance cost is  $M$ . Total cost is therefore

$$M + 3 \times \frac{n}{9}$$

so each member pays

$$\frac{1}{10} \left( M + 3 \times \frac{n}{9} \right).$$

Students with a stronger maths background will be able to simplify this to

$$\frac{M}{10} + \frac{n}{30}.$$

4. A spreadsheet might be,

	A	B	C	D	E
1	SOLD	MANUF.	18*SOLD	15*MANUF.	PROFIT
2			= 18*A2	= 15*B2	= C2 - (10 000 + D2)
3			= 18*A3	= 15*B3	= C3 - (10 000 + D3)
4			= 18*A4	= 15*B4	= C4 - (10 000 + D4)
⋮	⋮	⋮	⋮	⋮	⋮

etc.

where arbitrary values of the numbers sold and manufactured are entered into columns A and B.

It does not really make sense to consider values of numbers sold which are greater than the corresponding numbers manufactured but these give large profits.

To adapt the spreadsheet for *number sold = number manufactured*, replace A2, A3, A4, etc. in column C's formulae by B2, B3, B4, etc. respectively so that column A becomes redundant (or make column B redundant in a similar way). Some students may just make the values in column A identical to those in column B, which is cheating slightly, but will allow them to investigate the break-even point.

The values are

B	E
MANUF. = SOLD	PROFIT
1000	-7000
2000	-4000
3000	-1000
4000	2000
5000	5000

so it seems reasonable to deduce that 0 profit may be achieved when MANUF. = SOLD lies between 3000 and 4000. (It is actually at 3333.3.)

**ASSESSMENT 6**

The beginnings of algebra here, but don't tell them if they haven't realized!

1. a.  $3x + xy + 3y$

b.  $p - 2q - q^2 - 3p^2$

c.  $\frac{3}{2} \cdot \frac{a}{b} + a - \frac{a}{b} + 3a = \frac{1}{2} \cdot \frac{a}{b} + 4a$

2. a.  $\frac{6bf}{2de}$  (Students do not yet know how to cancel.)

b.  $3abcd$

3. No, it is not the same. Try for instance,

$a = 2:$        $b = 1:$        $d = 2$

then       $a(b + d) = 2(1 + 2) = 2 \times 3 = 6$

and       $ab + d = 2 \times 1 + 2 = 4.$