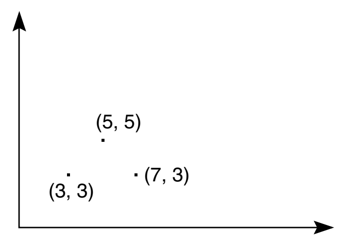
EM4 Modelling using straight lines

Students will like this chapter because they are generally happy drawing straight lines on graphs.

ASSESSMENT 1

Points on graphs, and we introduce the terminology of a function, linear function, linear equation and a coefficient. We consider the equation of a straight line.

1. a. NO



b. YES

2. a. YES **b.** YES **c.** NO **d.** No

e. YES.

f. Provided $x \neq 0$, we can multiply by x (although problems will occur at $x = 0$) to give $2y + x = 0$ which is linear.

g. YES.

3. These lines cross the axes at

a. $\left(0, \frac{3}{2}\right), \left(-3, 0\right)$

b. $\left(0, 2\right), \left(\frac{2}{5}, 0\right)$

c. (0, 8), (20, 0)

d. $(0, a), (b, 0)$

e. line passes through all points with x coordinate -5.

ASSESSMENT 2

The gradient and intercept of a straight line and its equation.

1. **a.** $\frac{1-2}{1-5} = \frac{-1}{-4} = \frac{1}{4}$

b. $\frac{5+1}{3+1} = \frac{6}{4} = \frac{3}{2}$

c. $\frac{-5+1}{6-7} = \frac{-4}{-1} = 4$

d. $\frac{-7+2}{-4-1} = \frac{-5}{-5} = 1$

2. **a.** intersect: $-\frac{1}{5}$ gradient: $-\frac{1}{4}$

b. intersect: $-\frac{2}{3}$ gradient: 12

c. intersect: 1 gradient: -1

d. intersect: $-\frac{c}{b}$ gradient: $\frac{a}{b}$

3. Rough sketches only, showing y-axis intersect and with suitable gradient are required.

a. intersect: 1 gradient: 3

b. intersect: -3 gradient: 3

c. intersect: -1 gradient: 3

d. intersect: -2 gradient: 3

4. Again rough sketches showing intersect and gradient only are required.

a. intersect: $\frac{2}{3}$ gradient: 2

b. intersect: 1 gradient: -7

c. intersect: -24 gradient: 12

ASSESSMENT 3

We introduce several applications including linear supply and demand functions and the concept of a feasible region, to be used in linear programming later.

1. $y = 1000 + 11x$

When $x = 0$ $y = 1000$
When $x = 500$ $y = 6500$
When the company spends A on advertising the average revenue is

$$100 + 11A$$

If they spend one more pound on advertising, i.e. spend $A + 1$, the average revenue will become

$$1000 + 11(A + 1) = 1000 + 11A + 11$$

So an extra £1 spent on advertising advantage increases average revenue by £11.

2. $Q = aP + b$
When $P = 1$, $Q = 990$ so

$$990 = a + b \qquad \text{(i)}$$

If P is one unit more, i.e. $P = 2$, we are told that $Q = 900$ so

$$900 = 2a + b \qquad \text{(ii)}$$

At this stage the students have not yet learned how to solve equations (i) and (ii) simultaneously. However, we hope they can see intuitively that the equations imply that a is -90, which means that b must be 1080. The demand equation is therefore

$$\text{Q} = -90P + 1080$$

This is a line with a negative slope which crosses the axes at (0, 1080) and (12, 0).

ASSESSMENT 4

Linear simultaneous equations in two unknowns. We cover all three types of solution and their graphical representation. Most students will have met this material before – or at least the unique solution case, although some will have developed their own way of doing things which will not extend to the infinite number or no solution case later and will need to be weaned off it.

1. a. $3x - 2y = 6$ multiplying by 3: $9x - 6y = 18$
$9x - 6y = 18$
the two equations respresent the same line, so there are an infinite number of solutions.

b. $5y + x = 6$ multiplying by 2:
$-10y - 2x = 9$

$$\begin{array}{r} 10y + 2x = 12 \\ -10y - 2x = 9 \\ \hline 0y + 0x = 21 \end{array}$$

the two lines are parallel, so there is no solution.

c. $22x - 11y = 7$
$11y + 20x = 35$
adding, term by term gives

$$\begin{aligned} 42x &= 42 \\ x &= 1 \\ 22 - 11y &= 7 \\ 11y &= 22 - 7 = 15 \\ y &= \frac{15}{11} \end{aligned}$$

ASSESSMENT 4

Sol.: $x = 1,\ y = \frac{15}{11}$, or $\left(1, \frac{15}{11}\right)$

d. $3x - 2y = 3$

$x + 3y = 12$ multiplying by 3: $3x + 9y = 36$

Subtracting: $11y = 33$

$y = 3$

$x = 12 - 3y = 12 - 9 = 3$

Sol.: $x = 3,\ y = 3$, or $(3, 3)$.

2. a. $ax - 2y = 2$

$x + 3y = 12$ multiplying by $\frac{2}{3}$: $\frac{2}{3}x + 2y = 2$

Adding term by term:

$$\left(a + \frac{2}{3}\right)x = 4$$

$$x = \frac{4}{a + \frac{2}{3}} = \frac{4}{\frac{3a+2}{3}} = \frac{12}{3a+2}$$

$$y = \frac{3-x}{3} = 1 - \frac{x}{3} = 1 - \frac{4}{3a+2}$$

$$= \frac{3a+2-4}{3a+2} = \frac{3a-2}{3a+2}$$

Sol.: $x = \frac{12}{3a+2},\ y = \frac{3a-2}{3a+2}$, or $\left(\frac{12}{3a+2}, \frac{3a-2}{3a+2}\right)$

b. $py + qx = 2$ multiplying by $\frac{q}{p}$: $qy + \frac{q^2}{p} = \frac{2q}{p}$

$qy - px = \frac{q^2 - p^2}{pq}$

Subtracting term by term:

$$\frac{q^2}{p}x + px = \frac{2q}{p} - \frac{q^2 - p^2}{pq}$$

$$x\left(\frac{q^2 + p^2}{p}\right) = \frac{2q^2 - q^2 + p^2}{pq}$$

$$x\left(\frac{q^2 + p^2}{p}\right) = \frac{q^2 + p^2}{pq}$$

$$x = \frac{q^2 + p^2}{pq} \cdot \frac{p}{q^2 + p^2} = \frac{1}{q}$$

$$y = \frac{2 - qx}{p} = \frac{2-1}{p} = \frac{1}{p}$$

Sol.: $x = \frac{1}{q},\ y = \frac{1}{p}$, or $\left(\frac{1}{q}, \frac{1}{p}\right)$