CHAPTER

15 Option Pricing

CHAPTER OUTLINE

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15.1 Introduction

In the previous chapter we examined some of the basic issues relating to options and looked at possible return profiles. In this chapter we look at the more complex question of option pricing, and in particular examine the factors that determine the price of an option, first intuitively and then analytically using the famous Black–Scholes option pricing formula which was put forward in a classic paper by Black and Scholes (1973). Although there have been many refinements to the Black–Scholes formula it has become one of the most famous equations of economics and is widely used by practitioners to determine appropriate option premiums. We also consider the relationship between call and put premiums via the put–call parity condition.

15.2 Principles of option pricing

When considering the price of an option we need to bear in mind exactly what the buyer of an option is purchasing. An option offers the purchaser limited downside loss as given by the option premium paid, combined with unlimited upside potential. An option that has no chance of ever being exercised would be worthless; however, if an option has a high probability of being exercised then one should expect to pay more for it. A fundamental principle underlying the pricing of an option is that the greater the probability of an option being exercised, the higher will be the option premium, other things being equal.

Bearing this basic principle in mind, let us consider conceptually the factors that are likely to influence the price of a European call option in company ABC. Payment for a European call option gives the buyer the right but not the obligation to buy shares in company ABC at a predetermined price at a given date in the future. There are five crucial factors that determine the likelihood of the call option being exercised and hence influence the price to be paid for the call option:

1 **The current price of the share** – the higher the current price of the stock the more likely the share is to be exercised for any given exercise price and consequently the higher the price of a call option.
2 **The strike price** – the higher the strike price of a call option the less likely it is that it will be exercised and hence the lower its price.
3 **The time left to expiration** – the longer the time left to expiration then the more the chance of the option being exercised and hence the higher its price.
4 **The volatility** – the more volatile an option is the more likely that its price will exceed the strike price at expiration and hence the higher the price of the option.
5 **The risk-free rate of interest** – the purchaser of a call option is paying the issuer cash for an option that can be exercised to buy an underlying security at a future date. The option holder is thus benefiting from the fact that the difference between the option premium and actually buying the underlying security can be invested at a risk-free rate of interest until the option expires. A rise in the risk-free rate of interest makes it more attractive to buy the option rather than the underlying security. For this reason, other things being equal, a call option premium needs to be priced more highly when interest rates are high than when interest rates are low. The higher the risk-free rate of interest the higher a call option price. Note, however, that changes in the risk-free rate of interest are usually only a marginal factor in the pricing of options.

Table 15.1 Summary of factors affecting an option's price

Factor a rise in:	European call	European put
Current price	+	–
Strike price	–	+
Time to expiration	+	+
Volatility	+	+
Risk-free interest rate	+	–

Note: The above assumes that there are no dividends due during the life of the option.

Table 15.1 summarizes the relationships between each of the five determinants of an option's price and the price of a European call and put option.

15.3 Intrinsic value and time value

An option premium is made up of two components, the **intrinsic value** and the **time value**. The intrinsic value is the gain that would be realized if an option was exercised immediately. For a call option, this is simply the strike price less the cash price of the underlying asset, while for a put option it is the strike price less the cash price:

Intrinsic value for a call option = Cash price – strike price

Intrinsic value for a put option = Strike price – cash price

If an intrinsic value for an option exists, then the option is said to be 'in-the-money'. A call option will be in the money if the strike price is below the cash price. If the strike price is above the cash price the call option will have zero intrinsic value and is said to be 'out-of-the-money'. If the strike price is equal to the cash price it is 'at-the-money' with zero intrinsic value. **Table 15.2** summarizes the various possible states for both call and put option contracts. The intrinsic value reflects the price that would be received if the option were 'locked in' today at the current market price, and is either positive or zero.

The **time value** of an option is the option premium less the intrinsic value, and reflects the fact that an option may have more ultimate value than the intrinsic value alone:

Time value = option premium – intrinsic value

Table 15.2 In-the-money, at-the-money and out-of-the-money

	Call	Put
In-the-money	Current price above exercise price	Current price below exercise price
At-the-money	Current price equals exercise price	Current price equals exercise price
Out-of-the-money	Current price below exercise price	Current price above exercise price

An option buyer, even if the option is out-of-the-money will still have some hope that at some time prior to expiration changes in the spot price will move the option into-the-money or further increase the value of the option if it is already in-the-money. This prospect gives an option a value greater than its intrinsic value.

NUMERICAL EXAMPLE

1 Consider a call option valued at 18 pence in the stock of company ABC with a strike price of 90 pence and a cash price for the underlying share of 100 pence. The option is in-the-money to the tune of 10 pence and so has an intrinsic value of 10 pence, while the other 8 pence represents the time value.

2 Consider a call option valued at 11 pence in the stock of company XYZ with a strike price of 85 pence and a cash price for the underlying share of 80 pence. The option is out-of-the-money and so has no intrinsic value; the whole value of the option, that is 11 pence, is time value.

15.4 The distribution of the option premium between time and intrinsic value

One of the crucial assumptions underlying the theory of option pricing as set out in the Black–Scholes option pricing formula is that the natural logarithm of the cash price of the underlying asset is normally distributed, that is it follows a log-normal distribution. A variable that has a log-normal distribution can have any value between zero and infinity as shown in **Figure 15.1.**

The time value for a given expiry date will get closer to zero the more in

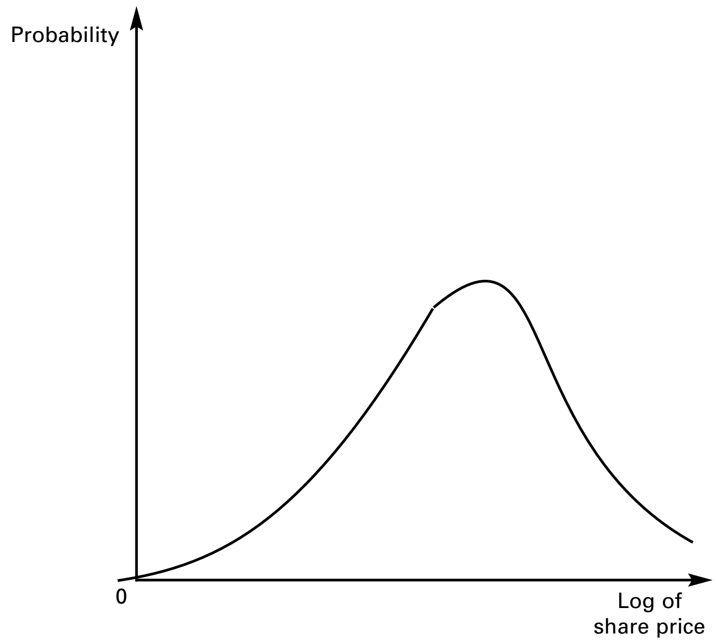


Figure 15.1 A log-normal distribution

the money or out-of-the-money the contract is. This is illustrated in **Figures 15.2a–e** that show the various probability factors behind the intrinsic and time value components of an option. There is a log-normal distribution around the spot price such that the spot price may go up or down by a given amount with equal probability; however, the larger the move in any direction the smaller the probability.

Figure 15.2(a) shows a deep-in-the-money option, with the spot price exceeding the exercise price. This option has a roughly 50 per cent chance of rising further giving good upside potential; however, there is also a lot of intrinsic value that could easily be reduced or even lost entirely if the security were to fall in price, which means that the time value is given by the potential upside area minus the potential loss of the intrinsic value which is quite high leaving a small amount of time value.

Figure 15.2(b) shows a slightly-in-the-money option, with the spot price exceeding the exercise price. This option has a roughly 50 per cent chance of rising further giving good upside potential, but there is less intrinsic value than in case **Figure 15.2a**, that could be wiped out or reduced, so the time value which is given by the potential upside area minus the potential loss of the intrinsic value is higher than in the case of **Figure15.2a** due to the lower intrinsic value.

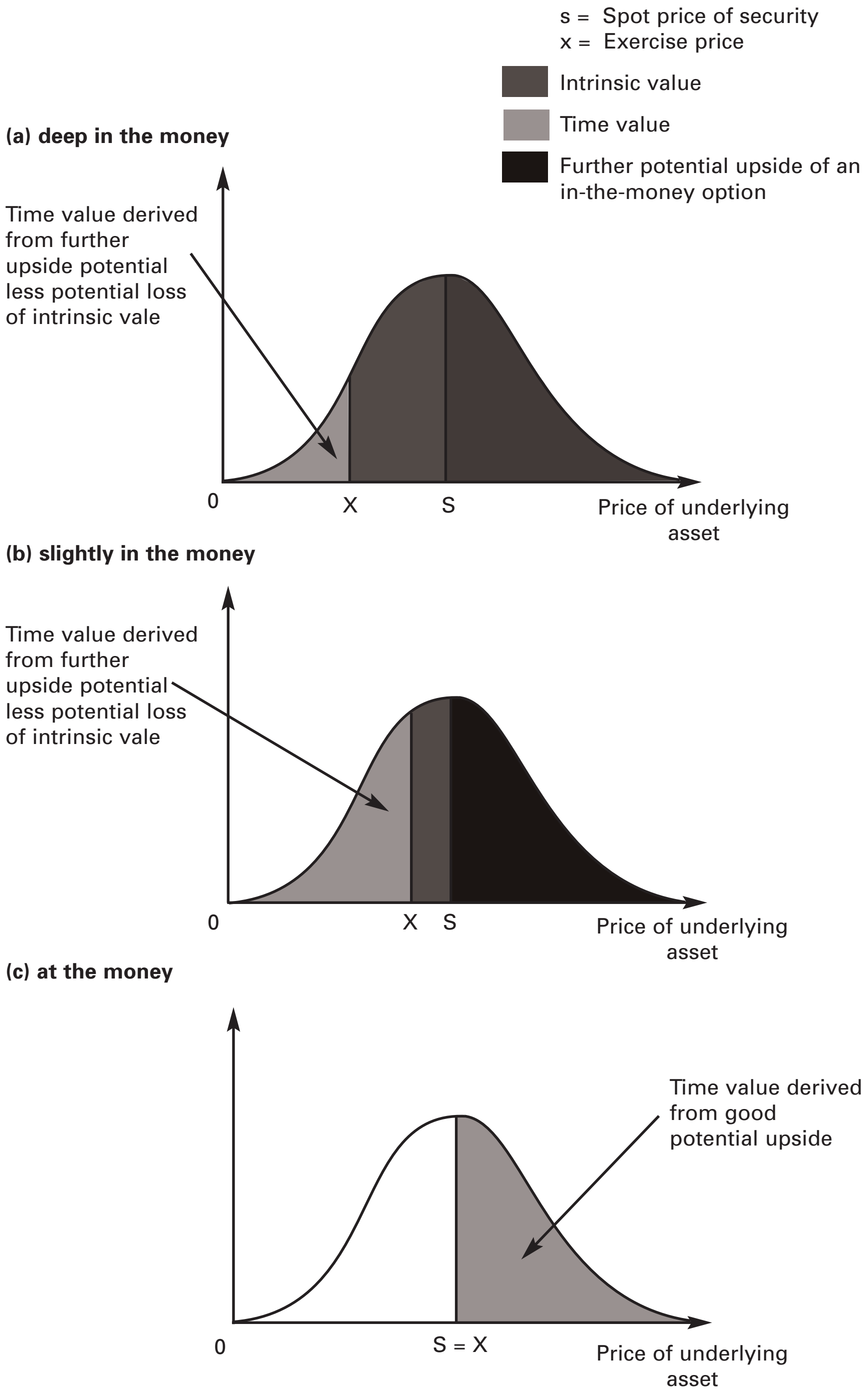
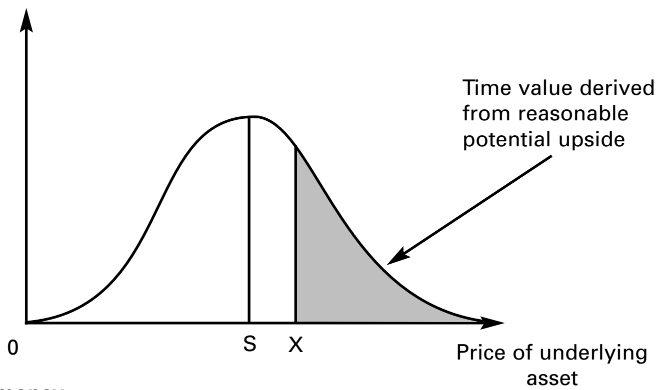


Figure 15.2 Intrinsic value and time value

(d) slightly out of the money



(e) deep out of the money

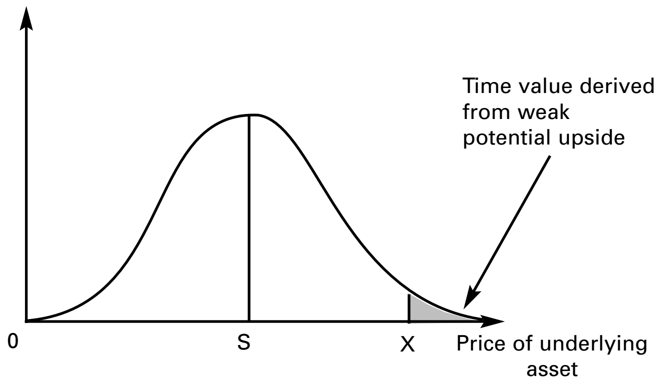


Figure 15.2 (*continued*)

Figure 15.2(c) shows an at-the-money option, with the spot price being equal to the exercise price. The time value of the option is at its maximum reflecting that any upward movement in the price of the underlying security will place the option in-the-money, while there is no intrinsic value to be lost.

Figure 15.2(d) shows a slightly-out-of-the-money option, with the spot price not too far below the exercise price. The option has no intrinsic value but there is a good chance that the spot price may exceed the exercise price prior to maturity, although less than in case 15.2(c). For this reason, the option will have a lower time value than in Figure 15.2(c), other things being equal.

Figure 15.2(e) shows a deep-out-of-the-money option, with the spot price well-below the exercise price. The option has no intrinsic value and there is only a relatively small chance that the spot price will exceed the exercise price prior to maturity. For this reason, the option has only a small time value which is lower than in case **15.2(d)**, other things being equal.

An important point about these examples is that the value of the option falls, other things being equal (that is for a given exercise price, volatility, risk-free rate of interest and term to maturity), as we move from **15.2(a) to (e)** since the probability of the option being exercised, that is the area to the right of the exercise price, decreases.

In **Figure 15.3**, the distribution of the total option premium between time value and intrinsic value is shown for a variety of spot prices (*S*1, *S*2, *S*3, *S*4 and *S*5), other things being equal and for a given strike price (*X*). A deep-out-of-the-money option with price *S*1 has zero intrinsic value and a small amount of time value. If the spot price were higher at *S*2 so that the option is only slightly out-of-the-money the option premium has zero intrinsic value but more time value reflecting the greater probability of being exercised than at *S*1. When the spot price *S*3 is equal to the exercise price, that is the contract is at-the-money, the entire premium is made up of the time value which is at its maximum. Above the exercise price, the option premium starts to have a positive intrinsic component which increases by 1 unit for each 1 unit the spot price exceeds the exercise price; however, time value starts to fall because

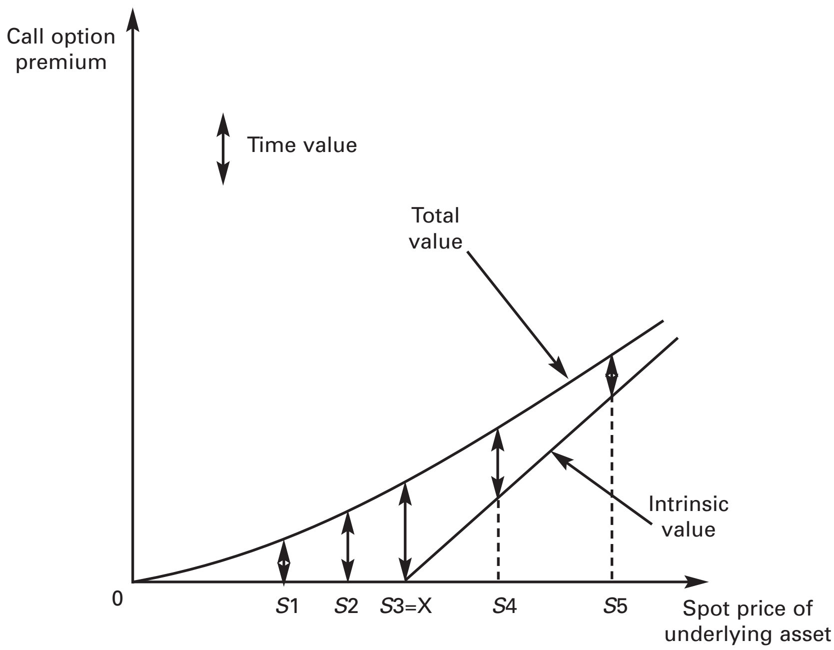


Figure 15.3 The distribution of a call premium between intrinsic and time value

Table 15.3 Intrinsic value and time value

Option status	Intrinsic value	Time value	Reason for time value
Deep-in-the-money (**Figure 15.2a**)	$S - X$	Low	Small downside protection
Slightly-in-the-money (**Figure 15.2b**)	$S - X$	High	High downside protection High upside potential
At-the-money (**Figure 15.2c**)	Zero	Maximum	Maximum upside potential Maximum downside protection
Slightly-out-of-the-money (**Figure 15.2d**)	Zero	High	High upside potential High downside protection
Deep-out-of-the-money (**Figure 15.2e**)	Zero	Low	Small upside potential

Note: S = cash or spot price of the underlying asset and X is the exercise price of the call option.

although there is further upside potential there is the risk that some (or all) intrinsic value can be lost, so at $S4$ the time value is smaller than at $S3$ although the total option premium is higher. At the spot price $S5$ the option is deep-in-the-money with a large component of intrinsic value and continued upside potential, and only a slight risk that the option will end up worthless which is reflected in a small amount of time value.

The lesson is that the more in the money the contract, the greater the probability that the option holder will be able to exercise the contract and therefore the lower the time value on the contract. Similarly, the more out of the money the contract the greater the probability that the contract will not be exercised and therefore the lower the time value of the options contract. **Table 15.3** summarizes the division of the option premium between intrinsic and time value for various option statuses, and the reason for the time value.

15.5 The Black–Scholes option pricing formula

In a famous paper, Fischer Black and Myron Scholes (1973) derived a formula for the pricing of options. The formula applies to European options although more sophisticated versions exist to deal with the pricing of American options. For the purpose of our analysis we will deal with the pricing of a call option. The Black–Scholes formula is based upon a number of simplifying assumptions:

1 The underlying asset being analysed pays no dividends or interest during its lifetime.
2 The option is a European option, that is it cannot be exercised prior to maturity.
3 The risk-free rate of interest is fixed during the life of the option.
4 The financial markets are perfectly efficient with zero transactions costs, no bid–ask spread and no taxes.
5 The price of the underlying asset is log-normally distributed, with a constant mean and standard deviation.
6 It is possible to short-sell the underlying asset and utilize the proceeds obtained without restriction.
7 The price of the underlying asset moves in a continuous fashion.

The basic idea underlying the derivation of the Black–Scholes option pricing model is that a long position in the underlying stock is neutralized by a short position in options (appropriately priced) such that the stock-holder with such a combined position will only have a return equal to the risk-free rate of interest. When the stock price rises, the premium on the option rises (implying a loss for a short position) so as to offset any gain from the rise in the price of the stock.

The starting point for the Black–Scholes formula is that the intrinsic value of a call option on expiration is the spot price (S) less the exercise price (X) if the option is in-the-money, or zero if the option is at or out-of-the-money. Imagine that we knew today with 100 per cent certainty the intrinsic value on expiration, and that this was above the exercise price, then the value of the call premium on expiration would be:

$$C = S - X > 0 \tag{15.1}$$

where C is the call premium; S is the cash or spot price of the underlying asset; and X is the exercise price.

The holder of such a call option will be able to set aside less money than the actual exercise price (X) prior to maturity, since during the time remaining to maturity (T) he can obtain a rate of interest r on such funds that when continuously compounded will give him the sum X when the exercise is due. The sum of money that needs to be set aside to achieve the sum X is given by:

$$Xe^{-rT} \tag{15.2}$$

where X is the exercise price; e is the natural number 2.7182 . . .; r is the risk-free rate of interest; and T is the time left to maturity expressed as a fraction of a year.

The term Xe^{-rT} is simply the present value of the exercise price when continuous time discounting is used. Hence, the value of a call option would actually be worth more than suggested by equation (15.1). At any time up to maturity, the value of such an option would be given by:

$$C = S - Xe^{-rT} \tag{15.3}$$

which says that the value of the call would be equal to the price of the share on expiration less the present value of the exercise price.

In reality, the assumption that the share price will close above the strike price is unrealistic. Hence, the actual present value of $S - Xe^{-rT}$ is uncertain, so equation (15.3) needs to be modified to be based upon the expected value upon expiration. The expected value involves use of the normal distribution tables, leading to:

$$C = S\,N(d1) - Xe^{-rT}\,N(d2) \tag{15.4}$$

$S\,N(d1)$ is the expected value of the underlying security upon expiration (assuming that the option is exercised), while the term $Xe^{-rT}\,N(d2)$ is the expected present value of the strike price on expiration (assuming that the option is exercised).

The $d1$ and $d2$ terms are given by:

$$d1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{15.5}$$

and

$$d2 = d1 - \sigma\sqrt{T} \tag{15.6}$$

where C is the price of the call; S is the current spot price; X is the exercise price; σ^2 is the variance of the price of the underlying asset on an annual basis; σ is the standard deviation of the price of the underlying asset on an annualized basis; and T is the time to expiry in a fraction of a year (e.g. one quarter = 0.25, 6 months = 0.5).

The Black–Scholes formula therefore states that the current value of a call option is the present value of the expected cash price less the expected value of the strike price.

Interpretation of the $N(d1)$ and $N(d2)$ terms

The $N(d1)$ and $N(d2)$ terms involving the cumulative probability function are the terms which take into account the risk of the option being exercised.

The $N(d1)$ term reflects the cumulative probability relating to the current value of the stock, and its value shows the amount by which the option premium increases for each 1 unit rise in the price of the underlying security. The value of $N(d1)$ lies between 0 and 1. If a stock is deeply out-of-the-money then any unit rise in the stock price will have little effect on the value of the call since it remains unlikely that the option will be exercised, so that $N(d1)$ will be low, for example 0.2. If the option is currently at-the-money, then there is a 50 per cent chance it will end up in-the-money and a 50 per cent chance it will end up out-of-the-money, so $N(d1)$ will be 0.5; that is, if the underlying stock rises by one unit then the option price will rise by 0.5 of a unit. If the option is already deep-in-the-money, each one unit rise in the share price will be increasingly reflected in the price of the call so that $N(d1)$ will get closer to 1, for example 0.9. The higher the stock price in relation to the exercise price, the higher the value of $N(d1)$.

The $N(d2)$ term is an approximate measure of the probability that the call option will actually be exercised; for example, if $N(d2)$ is 0.60 then there is an approximately (though not exactly) 60 per cent chance that the option will be exercised.

The value of $N(d1)$ is always greater than $N(d2)$ except in the special case when the option is certain to be exercised, in which case $N(d1) = N(d2) = 1$. When $N(d1)$ and $N(d2)$ are equal to 1 then the Black–Scholes option pricing formula becomes:

$$C = S - X\mathrm{e}^{-rT} \tag{15.7}$$

One of the most notable features of the Black–Scholes option pricing formula is that expected volatility is a key factor in determining the price of an option, the formula does not depend upon the level of the future price of the underlying asset to determine the appropriate option price.

NUMERICAL EXAMPLE

Let us consider the pricing of a call option for shares in company ABC. For simplicity, we ignore complications posed by the possibility of dividend payments. Let us assume that the current spot price of a share is 100 pence and an investor buys a call option to purchase the share at 90 pence. The risk-free rate of interest is 6 per cent and the relevant measure of historical volatility is 49 per cent. The option has 90 days to expiry. Hence:

S = 100p
X = 90p
r = 0.06
T = 90/365 = 0.25 (approx)
σ^2 = 0.49 so that σ = 0.7

We first calculate the value of d1:

$$d1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

that is,

$$d1 = \frac{\ln(100/90) + (0.06 + 0.7^2/2)0.25}{0.7\sqrt{0.25}} = 0.52$$

From the cumulative normal distribution **Table 15.8** supplied at the end of the chapter we find:

$d1$ = N(0.52) = 0.6985

Note: If the value we find for $N(d1)$ or $N(d2)$ is negative we must subtract the value we find in the table from 1. For example, if we need to look up the value $N(d1) = N(-0.12)$ in the table, and we first look up 0.12 which is 0.5478, and then subtract this from 1 so that $N(d1) = N(-0.12)$ is given by 1 – 0.5478 = 0.4522. For positive values of $N(d1)$ and $N(d2)$ we use the value listed in the table.

We next calculate the value of $d2$:

$$d2 = d1 - \sigma\sqrt{T}$$

that is,

$$d2 = 0.52 - 0.7\sqrt{0.25} = 0.17$$

With these calculations we are now in a position to calculate the price of the option:

$$C = S\,N(d1) - Xe^{-rT}\,N(d2)$$

Substituting the appropriate values yields:

$$C = 100\,(0.6985) - 90(2.718)^{-0.06(0.25)}\,(0.5675) = 19.35$$

Of the 19.35 premium 10 pence is intrinsic value and 9.35 pence is time value.

The above calculations are based upon a current share price of 100 pence and a strike price of 90 pence. **Table 15.4** shows how the values of *N*(*d*1) and *N*(*d*2) change as the value of the current share price changes, and the resulting call price in pence.

Table 15.4 The values of *N*(*d*1) and *N*(*d*2) for different current share prices

Share price in pence	*d*1	*N*(*d*1)	*d*2	*N*(*d*2)	Call price in pence
70	–0.50	0.3085	–0.85	0.1977	4.07
80	–0.12	0.4522	–0.47	0.3192	7.88
90	0.22	0.5871	–0.13	0.4483	13.08
100	0.52	0.6985	0.17	0.5675	19.35
110	0.79	0.7852	0.44	0.6700	26.97
120	1.04	0.8508	0.69	0.7549	35.18
130	1.27	0.8980	0.92	0.8212	43.93

Notes: T = 3 months (0.25 in formula), r = 6 per cent (0.06 in formula), standard deviation = 0.7, X = 90 pence.

15.6 Different measures of volatility

The intrinsic value of an option is easily calculated, and the time left to expiration and the risk-free rate of interest are all measurable; the most contentious thing to measure is volatility. Ideally, the efficient pricing of options would require a measurement of volatility that is likely to reflect the volatility that will occur in the future. **Historical volatility** may be a useful measure for this purpose, but it could prove to be defective as the past is not necessarily a good guide to the future. In addition, should an appropriate measure of historical volatility be based on the last month, the last three months, the last 6 months or last year? Another problem with historical volatility is that it usually fails to pick up the possibility of large discrete shifts. **Expected volatility** will differ from one market participant to another and therefore the view of the appropriate market price of an option will vary between market participants. **Implied volatility** is the volatility implicit in the current option price, found by taking the current price of the option and finding a volatility that when plugged into the option-pricing formula gives the current market price of the option (this is easily calculated using a spreadsheet).

15.7 The calculation of historical volatility

Volatility in the Black–Scholes option pricing formula can be measured by historical volatility, the most common method of calculation being the annualized standard deviation of daily, weekly or even monthly changes in prices. The annualized price volatility is obtained by multiplying the calculated sample standard deviation by the number of periods:

For daily data (based on 252 trading days per annum):

$$\sigma = \sqrt{252} \times \text{daily standard deviation}$$

For weekly data:

$$\sigma = \sqrt{52} \times \text{weekly standard deviation}$$

For monthly data:

$$\sigma = \sqrt{12} \times \text{monthly standard deviation}$$

The volatility used is therefore the annualized standard deviation of the changes in prices, which are most easily calculated by taking the natural log of relative prices as in the example shown in **Table 15.5**.

From the table, the annual standard deviation = $0.0975 \times \sqrt{52} = 0.7$ and the annual variance = $0.7 \times 0.7 = 0.49$. The correct variance estimator in the Black–Scholes model is the annual variance of relative log prices.

Table 15.5 Example calculation of volatility

Week	Share price (S_t)	Relative price (S_t/S_{t-1})	Log of relative prices i.e. $\ln(S_t/S_{t-1})$
1	91		
2	102	1.1209	0.1141
3	95	0.9314	–0.0711
4	101	1.0632	0.0612
5	116	1.1485	0.1385
6	101	0.8707	–0.1385
7	108	1.0693	0.0670
8	95	0.8796	–0.1283
9	102	1.0737	0.0711
10	107	1.0490	0.0479

Note: The standard deviation of the log of relative prices is 0.0975.

15.8 Problems with the Black–Scholes option pricing formula

The formula we have looked at is only applicable to European options. American options are usually priced slightly higher than European options because of the extra advantage that they give to the holder of being able to exercise the option at any date prior to maturity.

Another consideration is that the formula assumes that the log of the share price follows a log-normal distribution, whilst the real world distributions tend to be leptokurtic, that is to have fatter tails than a normal distribution reflecting that there are better chances of an option being exercised than suggested by the Black–Scholes formula. Hence, real world option prices tend to exceed the Black–Scholes formula price.

15.9 The sensitivity of options prices

The Black–Scholes option pricing formula shows that the price of options is determined by the time left to maturity, the strike price, the risk-free rate of interest, the volatility of the underlying share and its price. Any fund manager or investor using options will be interested in how the price of an option is affected by changes in any of the above listed factors, we briefly mention these measures:

- **Option theta (θ)**. An option's theta measures an option's sensitivity to the passing of time. The longer an option has until expiry the greater the possibility of time value being realized. The time value of an option will fall over time according to the square root of time.
- **Option delta (δ)**. An option's delta measures the sensitivity of an option's price to the price of the underlying share. The formula for a call option is given by:

  $$C = S\,N(d1) - Xe^{-rT}\,N(d2)$$

  and the delta for a call option is given by:

  $$\Delta C = \frac{\partial C}{\partial S} = N(d1) \leq 1$$

The value of delta on a long call or short put option will lie between 0 and 1. Delta is a particularly important measure, since its inverse yields what is known as the riskless hedge ratio, that is a ratio of calls that need to be sold to protect a position in the underlying stock. For example if delta is $N(d1) = 0.6985$, then the hedge ratio $h = 1.432$. Given that a standard option contract is for 1,000 shares, to hedge these 1,000 shares it would be necessary to write 1.432 option contracts.

From our example, for a share currently priced at 100 pence with a strike price of 90 pence and a current premium of 19.35 pence per option, each time the share rises by 1 pence the holder of the underlying share makes 1 pence on holding the underlying share. On the other hand, the holder will lose $1.432 \times 0.6985 = 1$ pence from a loss on writing the call option contract.

- **Option gamma (γ).** This a measure of the rate at which an option's theta is changing. It is given by the change is delta divided by the change in the share price. If a share price moves from 100 pence to 101 pence and this causes the delta on the 90 pence call option to move from 0.69 to 0.70 then the gamma on a 90 pence call is 0.01/1 = 0.01.
- **Option lambda (λ) or option kappa (κ).** An option's lambda measures the sensitivity of an option's price to changes in the underlying volatility of the share. That is the change in the call premium divided by the change in the variance of the share price.
- **Option rho (ρ).** An option's rho measures the sensitivity of an option's price with respect to a percentage change in the interest rate. That is, the change in the call premium divided by the percentage change in the interest rate.

15.10 Put–call parity

Calls and put options, while they offer very different rights, are nonetheless linked together via a fundamental arbitrage relationship. This relationship, described by Stoll (1969), is known as the put–call parity. The relationship holds only for European options.

The basis of the put–call parity relationship is that combining a long position in the security with both a short call and long put contract with the same exercise price X and expiry date T creates a riskless hedge portfolio, that is, a portfolio with a known guaranteed value in the future. Since the portfolio will have a known guaranteed value, then the return on the portfolio should be no greater than the current risk-free rate of interest.

If the contract expires in-the-money, then the investor will have:

value of security = S
+ value of long put = 0
– value of short call = $X - S$

value of portfolio = X

Hence, the value of the portfolio is X.

If on the other hand the contract expires out-of-the-money then the investor will have:

value of security = S
+ value of long put = $X - S$
– value of short call = 0

value of portfolio = X

Hence, the value of the portfolio is X.

Table 15.6 illustrates how a combination of the underlying security and a short call and long put position at a given strike price will result in a riskless hedge portfolio with a known guaranteed future value equal to the exercise price regardless of what happens to the price of the security.

Since such a portfolio is riskless, the value of the portfolio at the time of its construction must be X discounted by the riskless rate of interest, that is Xe^{-rT}.

$$S + P - C = Xe^{-rT} \tag{15.8}$$

where S is the spot price; P is the put premium; and C is the call premium.

So that:

$$P = C - S + Xe^{-rT} \tag{15.9}$$

This means that once we have calculated the call premium via the Black–Scholes option pricing formula, then it is simple to also calculate the relevant put option price.

Table 15.6 Creation of a riskless hedge portfolio

Price	Value of short call position at expiry	Value of long put position at expiry	Value of security + short call + long put
70	0	20	90
80	0	10	90
90	0	0	90
100	–10	0	90
110	–20	0	90
120	–30	0	90
130	–40	0	90

In our example, given the following parameters:

$S = 100$, $X = 90$, $r = 0.06$, $T = 90/365 = 0.25$ (approx), $\sigma^2 = 0.49$ so that $\sigma = 0.7$

We have previously calculated that the call price is 19.35 pence. The appropriate put price is therefore:

$$P = C - S + Xe^{-rT}$$
$$= 19.35 - 100 + 90\ (2.7182)^{(-0.06\times 0.25)}$$
$$= 8.01 \text{ pence}$$

The put premium in this case is below the call premium. To check that this is an appropriate price the investor should be left with the discounted strike price regardless of what happens to the price of the share as depicted in **Table 15.7**.

Since a portfolio with a current share price of 100 pence, a strike price of 90 pence, a short call option of 19.35 pence and a long put of 8.01 pence is entirely riskless whatever happens to the share price, the future value of this portfolio should be equal to the riskless rate of interest of 6% (0.06). The holder of a share which is currently priced at 100 pence should end up with a portfolio worth only 1.5 per cent more upon expiry (3 months); that is the security would have risen to 101.50, and this is the case as shown in **Table 15.7**.

In the table, the profit on a short call (that is writing the call option) is calculated as the option premium 19.35 plus the risk-free rate of interest receivable by investing the premium at the end of three months (19.35 ×

Table 15.7 Creating a risk-free portfolio

Price of security expiry	Profit on short call at expiry	Profit on long put at expiry	Net value of security + short call + long put at expiry
70	19.64	11.87	101.51
80	19.64	1.87	101.51
90	19.64	–8.13	101.51
100	9.64	–8.13	101.51
110	–0.36	–8.13	101.51
120	–10.36	–8.13	101.51
130	–20.36	–8.13	101.51

1.015 = 19.64 pence) less the value of the call at expiry. For example, if the price of the security on expiration is 70 pence, the profit on the short call is 19.35 pence plus 1.5 per cent interest minus zero = (19.35 × 1.015) = 19.64 pence, whereas if the current price of the security is 120 pence, the profit on the call is 19.35 pence plus 1.5 per cent interest minus 30 pence = (19.35 × 1.015) – 30 = –10.36 pence. The profit on the long put is the value of the put at expiry less the put premium paid 8.01 less foregone interest on the put at the end of three months (8.01 × 1.015 = 8.13 pence). For example, if the price of the security on expiry is 70 pence, the profit on the long put is 20 pence minus 8.13 pence (8.01 × 1.015), that is 11.87 pence, whereas if the price of the security at expiry is 120 pence the profit on the long put is 0 pence minus 8.13 pence (8.01 × 1.015). As we can see in **Table 15.7**, whatever the price of the security upon expiry the combination of a the underlying share at 100 pence and the short call position and long put leaves the portfolio worth 101.51 pence regardless of the share price at expiry.

15.11 Conclusions

Option pricing is a relatively complex area and there are some crucial assumptions that need to be made for a valid application of the Black–Scholes option pricing formula. In particular, the expectation that the underlying asset on which the option is based has a log-normal distribution. In the market, the formula also needs to be amended to take account of problems such as dividend payments, whilst perhaps the biggest problem facing the formula concerns the appropriate volatility to be used; there is no guarantee that any historical volatility measure will be a fair approximation of the likely future volatility of the underlying asset. The market price of an option can be used to solve for the implied volatility, and participants that think that the implied volatility is inappropriate can write/sell options accordingly in the hope a making a profit. This ability to take a position on the likely volatility of a financial asset (for example, a straddle position) is just one of the innovative strategies that options permit for financial market participants.

There are many factors that interplay in the appropriate pricing of an option, including the risk-free rate of interest, the strike price, the spot price of the underlying instrument, its volatility, and the time left to expiry of the option. The beauty and significance of the Black–Scholes option pricing formula is the way that all these factors are brought together, and it is no exaggeration to say that it is one of the most important and most widely applied economics formulas in the real world! The put–call parity formula shows that there is a clear arbitrage relationship between the call and put premiums, and any divergence from this relationship will lead to arbitrage on the part of market participants.

Table 15.8 The cumulative distribution function for the standard normal random variable

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7781	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8897	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

MULTIPLE CHOICE QUESTIONS

1 Other things being equal, which of the following will cause the price of a call option on shares to increase?
- a A higher exercise price ☐
- b Lower interest rates ☐
- c A fall in the volatility of the share ☐
- d A longer time to expiry ☐

2 An options theta measures the sensitivity of an option premium to:
- a a change in the price of the underlying share ☐
- b a change in the underlying volatility of the share ☐
- c a change in the interest rate ☐
- d the passing of time ☐

3 A share has a current price of 100 pence, a call premium for a strike price of 80 pence is 40 pence, the risk-free rate of interest is 4% and expiry is in six months' time. According to put–call parity, what is an appropriate value for the put premium for a strike price of 80 pence and six months to expiry?
- a 19.8 pence ☐
- b 29.8 pence ☐
- c 39.8 pence ☐
- d none of the above ☐

4 Other things being equal, which of the following will cause the price of a put option to fall?
- a A higher exercise price ☐
- b A higher price of the underlying share ☐
- c A rise in the volatility of the share ☐
- d A longer time to expiry ☐

5 An option's delta measures the sensitivity of an option premium to:
- a a change in the price of the underlying share ☐
- b a change in the volatility of the share ☐
- c a change in the interest rate ☐
- d the passing of time ☐

6 A share has a current price of 100 pence, a call premium for a strike price of 90 pence is 19.35 pence, the risk-free rate of interest is 6% and expiry is in three months' time. According to put–call parity, what is an appropriate value for the put premium for a strike price of 90 pence and three months to expiry?
- a 6 pence ☐
- b 8 pence ☐
- c 10 pence ☐
- d None of the above ☐

7 Other things being equal, which of the following will cause the price of a call option to fall?
- a A lower exercise price ☐
- b Higher interest rates ☐
- c A rise in the volatility of the share ☐
- d A shorter time to expiry ☐

8 You are given the following data on call and put premiums in pence per share for company ABC shares, which are currently priced at 311 pence. Each contract refers to 1,000 shares:

	Call premiums in pence		
Strike prices	*April*	*June*	*September*
300 pence	31	49	61
330 pence	18	35	48

	Put premiums in pence		
Strike prices	*April*	*June*	*September*
300 pence	20	33	44
330 pence	36	50	60

Which of the following statements is **false**?

a The time value for the September 300 pence call premium is higher than for the September 300 pence put premium. ☐

b The intrinsic value for the June 300 pence call premium is the same as for the September 300 pence call premium. ☐

c The intrinsic value for the June 300 pence call premium is higher than for the June 330 pence put premium. ☐

d The time value for the April 300 pence put premium is higher than the intrinsic value for the April 330 pence put premium. ☐

SHORT ANSWER QUESTIONS

1 You are given the following data on call and put premiums in pence per share for company ABC shares, which are currently priced at 425 pence. Each contract refers to 1,000 shares.

	Call premiums in pence			*Put premiums in pence*		
Strike prices	*April*	*June*	*September*	*April*	*June*	*September*
420 pence	22	31	34	14	20	27
460 pence	6	12	15	39	42	48

(i) List all the call and put premiums that are 'out of the money'.

(ii) Explain the intuition as to why the premiums rise between April and September.

(iii) Which of the above options has the lowest time value?

(iv) Explain what you would do using any one of the above premiums if you expect the share price to fall to 300 pence by expiry, and the total profit you will make measured in pounds if you are proved correct?

2 You are given the following information about the stock of Company A:

Share price \$60, risk-free rate of interest 8%, time to expiration 3 months, annualized standard deviation 0.4, and exercise price \$65.

(i) Calculate the appropriate call premium on the stock according to the Black–Scholes option pricing formula. Show your workings in full.

(ii) Calculate an appropriate put premium. Show your workings in full.

3 Briefly discuss the relationship between a call premium and the five determinants of the call premium according to the Black–Scholes option pricing model.

4 (i) Explain the difference between historical volatility and expected volatility and their potential significance for option pricing.

(ii) Explain why an option writer is prepared to 'write' call options even though the potential losses are large compared to the potential premium to be received.

(iii) What does a rise in **implied volatility** potentially signify?

5 State which of the following statements are **true** and which are **false**:

(i) A call premium for a strike price of 200 pence is 15 pence, while the share is currently priced at 190 pence. The time value for the call premium is greater than the intrinsic value.

(ii) The Black–Scholes model provides an estimate of the price of an American option on a dividend-paying stock.

(iii) If implied volatility rises, other things being equal, both call and put premiums will rise.

(iv) If on a newly issued option the share price is 100 pence and the strike price is 100 pence, then time value will be at its maximum.

Further reading

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