

4. EXERCISES ON ESSENTIAL MATHEMATICS (Textbook p436)

A. ALGEBRA

A1. Rearranging an equation (p4)

Q1 Rearrange each of the following equations and hence find the value of y ;-

(a) $3y = 15$, (b) $2y + 5 = 9$, (c) $\frac{y}{4} = 6$, (d) $\frac{5y}{3} = 35$, (e) $\frac{2y - 3}{3} = 5$

Q2 Rearrange each of the following equations to make z the subject of the equation;-

(a) $yz = 8$, (b) $3z - 2w = 6$, (c) $\frac{z}{p} = 5$ (d) $\frac{az}{b} = c$

Q3 (a) Rearrange $\rho = \frac{m}{V}$ to make (i) m (ii) V the subject of the equation,

(b) Rearrange $C = 2\pi r$ to make r the subject of the equation,

(c) Rearrange $A = \frac{\pi d^2}{4}$ to make d^2 the subject of the equation

(d) Rearrange $T^2 = \frac{4\pi^2 L}{g}$ to make L the subject of the equation.

[Click here for answers to A1](#)

A2. Using the dynamics equations (p136)

Q1 (a) Rearrange $v = u + at$ to make (i) a the subject, (ii) t the subject.

(b) Rearrange $s = \frac{(u + v)t}{2}$ to make (i) t the subject , (ii) v the subject .

Q2 (a) Rearrange $s = ut + \frac{1}{2}at^2$ with $u = 0$ to make a the subject.

(b) Rearrange $v^2 = u^2 + 2as$ to make (i) s the subject, (ii) a the subject

Q2 (a) Combine $v = u + at$ and $s = \frac{(u + v)t}{2}$ to eliminate u

(b) Hence show that $s = vt - \frac{1}{2}at^2$

Q3 A rocket launched vertically from the ground accelerates at constant acceleration a for time t_1

before the rocket motors cut off. Show that

(a) its speed when the motors cut off , $v = at_1$

(b) its height at the instant the motors cut off , $h = \frac{1}{2} at_1^2$

(c) its maximum height $H = \frac{1}{2} \left(1 + \frac{a}{g} \right) a t_1^2$

[Click here for answers to A2](#)

A3. Pythagoras theorem (p117)

[Fig 4A Trigonometry](#)

Q1 Calculate h , given (a) $o = 3$, $a = 4$ (b) $o = 5$, $a = 12$ (c) $o = 9$, $a = 40$

Q2 Calculate o given (a) $h = 13$, $a = 5$, (b) $h = 5$, $a = 3$ (c) $h = 41$, $a = 9$

Q3 Calculate a given (a) $h = 10$, $o = 8$, (b) $h = 39$, $o = 15$ (c) $h = 205$, $o = 45$

[Click here for answers to A3](#)

A4. Simultaneous equations (p190)

Q1 Solve the following pairs of equations for y and z :-

(a) $2y + 3z = 7$, $2y + 4z = 8$ (b) $y + 2z = 6$, $2y + z = 9$

Q2 Solve the following pairs of equations for p and q :-

(a) $4p - 2q = 16$, $3p + q = 17$ (b) $5q - 3p = 11$, $p + 2q = 11$

Q3 Solve the following equations for E / volts and r / ohms :-

(a) $E = 2r + 4$, $E = r + 8$ (b) $2E - 4r = 2$, $E - r = 2$

[Click here for answers to A4](#)

B. CALCULUS

B1. Rates of change in dynamics (p141)

Q1 An object travels along a straight line without change of direction. Its distance, s (in metres), from a fixed point on the line changes with time t (in seconds) in accordance with the equation

$$s = 10t + 6$$

(a) Complete the following table:-

t / seconds	0	1.0	2.0	3.0	4.0
s / metres	6.0	16.0			

(b) Calculate the distance moved Δs from (i) 0 to 1.0 s, (ii) 1.0 to 2.0 s, (iii) 2.0 to 3.0 s.

(c) Plot a graph to show how the distance, s , changed with time t .

(d) Show that the speed is constant and find its value.

Q2 An object rolls down a slope without change of direction. Its distance, s (in metres) from the bottom of the slope changes with time t (in seconds) in accordance with the equation

$$s = 2.5 - 0.10t^2$$

(a) Complete the following table :-

t / seconds	0	1.0	2.0	3.0	4.0	5.0
s / metres	2.5	2.4	2.1	1.60		

(b) Calculate the distance moved Δs from (i) 0 to 1.0 s, (ii) 2.0 to 3.0 s, (iii) 4.0 to 5.0 s.

(c) Plot a graph to show how the distance, s , changed with time t .

(d) Use your calculations in (b) to explain how the speed of the object changed from 0 to 5 s.

Q3 The speed , v (in m s^{-1}) of an object rolling down a slope changed with time t (in seconds) in accordance with the equation

$$v = 12 + 4 t$$

(a) Complete the following table ;-

t / seconds	0	1.0	2.0	3.0	4.0	5.0
v / m s^{-1}	12					

(b) Calculate the increase of speed Δv from (i) 0 to 1.0 s, (ii) 2.0 to 3.0 s, (iii) 4.0 to 5.0 s.

(c) Plot a graph to show how the speed , v , changed with time t .

(d) Hence show that the acceleration of the object is 4 m s^{-2} .

[Click here for answers to B1](#)

B2. Rates of change in exponential decay ;

Capacitor discharge (p214)

Q1 The charge Q (in μC) on a capacitor decays with time t (in seconds) in accordance with the equation

$$Q = 100 e^{-0.4t}$$

(a) Complete the following table ;-

t / seconds	0	1.0	2.0	3.0	4.0	5.0
Q / μC	100					

(b) Calculate the loss of charge ΔQ from (i) 0 to 1.0 s, (ii) 2.0 to 3.0 s, (iii) 4.0 to 5.0 s.

(c) Hence calculate the average current in the interval from (i) 0 to 1.0 s, (iii) 4.0 to 5.0 s.

Q2 The potential difference V (in V) across a capacitor decays with time t (in seconds) in accordance with the equation when the capacitor is discharged through a $2.2\text{ k}\Omega$ resistor .

$$V = 12 e^{-0.3 t}$$

(a) Complete the following table ;-

t / seconds	0	1.0	2.0	3.0	4.0	5.0
V / V	12					

(b) Plot a graph to show how the potential difference decreases with time .

(c) Use your graph to determine the time taken for the p.d. to fall to 0.37 of its initial value and hence determine the value of the capacitance.

[Click here for answers to B2](#)

Radioactive decay (p333)

Q1 The number of nuclei N of a sample of a radioactive isotope decreases with t (in seconds) in accordance with the equation

$$N = N_0 e^{-0.4 t} \quad \text{where } N_0 = 5.0 \times 10^{20}.$$

(a) Complete the following table ;-

t / seconds	0.0	1.0	2.0	3.0	4.0	5.0
N	5.0×10^{20}					

(b) Plot a graph to show how N decreases with time .

(c) Use your graph to determine the half life of the sample.

[Click here for answers to Radioactive decay](#)

B3. Differentiation of x^n and e^x (p215 and p336)

Q1 Use the differentiation rule $\frac{d}{dx}(x^n) = n x^{n-1}$ to differentiate each of the following;-

(i) $2x^3$, (ii) $4x^5$, (iii) $x^3 - 2x$, (iv) $4x - 3x^2$, (v) $\frac{1}{3}x^4$

Q2 Use the differentiation rule $\frac{d}{dx}(e^{kx}) = k e^x$ to differentiate each of the following ;-

(i) $2e^x$ (ii) e^{3x} (iii) e^{-x} (iv) $3e^{-x}$ (v) e^{-2x}

Q3 Differentiate each of the following functions for s with respect to t to give an expression for

$\frac{ds}{dt}$ in terms of t ;-

(i) $s = 5t + 3t^2$, (ii) $s = 4 + 2t - 6t^2$, (iii) $s = 3t - 5t^3$, (iv) $s = 4t^3 - 2t^2$

Q4 For constant acceleration, the displacement of an object $s = ut + \frac{1}{2}at^2$ where u is the initial velocity of the object. Show that

(a) by differentiating s with respect to t , the velocity at time t , $v = u + at$,

(b) by differentiating v with respect to t , the acceleration of the object = a .

[Click here for answers to B3](#)

B4. Differentiation of a sine function (p293)

Q1 Use the differentiation rule $\frac{d}{dt}(\sin bt) = b \cos bt$ to differentiate each of the following expressions ;-

(i) $\sin 2t$, (ii) $3 \sin t$, (iii) $\sin 0.5t$, (iv) $2 \sin 0.5t$, (v) $3 \sin 5t$

Q2 Use the differentiation rule $\frac{d}{dt} (\cos bt) = -b \sin bt$ to differentiate each of the following expressions ;-

(i) $\cos 3t$, (ii) $2 \cos t$, (iii) $\cos 0.1t$, (iv) $3 \cos 4t$, (v) $0.5 \cos 4t$

Q3 For a moving object with a displacement s from a fixed point, its velocity $v = \frac{ds}{dt}$ and its acceleration $a = \frac{dv}{dt}$.

Use the above differentiation rules to show that

(i) the velocity of an object $v = \omega r \cos \omega t$, if its displacement s varies sinusoidally with time in accordance with the equation $s = r \sin \omega t$.

(ii) the acceleration , a , of the object in (i) is given by the equation $a = -\omega^2 s$

[Click here for answers to B4](#)

C. GRAPHS

C1. Straight line graphs (p92, p96, p139)

Q1 Write down (a) the gradient , (b) the y-intercept for each of the following straight line equations ,

(i) $y = 2x$, (ii) $y = 3x + 5$, (iii) $y = 6 - 2x$, (iv) $y = 3 + 2x$, (v) $y = \frac{3}{2}x - 4$

Q2 (a) Plot each of the following equations on a graph covering the range from $x = -2$ to $x = +2$;-

(i) $y = 4x + 1$, (ii) $y = 2x + 3$

(b) Show that the two lines intercept each other at the point $x = 1$, $y = 5$

Q3 The speed v (in m s^{-1}) of an object varies with time t (in s) in accordance with the equation

$$v = 20 - 4t$$

(a) Sketch a graph to show how the speed varies with time,

- (b) Show that
- (i) the speed of the object at $t = 0$ is 20 m s^{-1}
 - (ii) the acceleration of the object = -4.0 m s^{-2} .

[Click here for answers to C1](#)

C2. Inverse square law (p239 , p330 , p398)

Q1 The force F (in newtons) between two ions varies with their distance apart r (in metres) in accordance with the equation

$$F = \frac{k}{r^2}, \text{ where } k = 2.0 \times 10^{-27} \text{ N m}^2$$

(a) Complete the following Table ,

$r / 10^{-9} \text{ m}$	0.50	1.0	1.5	2.0	2.5	3.0
F / N		2.0×10^{-9}				

(b) Plot a graph to show the variation of force with distance from 0.5 to 3.0 nanometres.

Q2 A geiger counter was used to measure the count rate due to a source of gamma radiation. The count rate C (in counts per second) varied with distance r (in metres) from the source in accordance with the equation

$$C = \frac{2.0}{r^2}.$$

- (a) Calculate the count rate at a distance of (i) 0.010 m , (ii) 0.020 metres from the source ,
- (b) The background count rate was 0.4 counts per second. Calculate the distance from the source at which the count rate due to the source would equal the background count rate.

Q3 The force , in newtons, between a comet and the Sun varies with distance (in metres) from the centre of the Sun in accordance with the equation

$$F = \frac{k}{r^2}, \text{ where } k = 4.0 \times 10^{32} \text{ N m}^2.$$

(a) Complete the following Table ,

$r / 10^9\text{m}$	0.50	1.0	1.5	2.0	2.5	3.0
F / N		4.0×10^{14}				

(b) Plot a graph to show the variation of force with distance from 0.5 to 3.0×10^9 m.

[Click here for answers to C2](#)

C3. Rates of change

dynamics - see Calculus ; [Rates of change in dynamics](#)

exponential decay - see Calculus ; [capacitor discharge](#)

Calculus ; [radioactive decay](#)

C4. Area under a curve (p100, p210, p403)

Q1 The graph below shows how the tension T in a spring varies with its extension, e .

[Fig 4B Tension v extension for a spring](#)

(a) Determine the spring constant k .

(b) (i) Use the graph to determine the energy stored in the spring when its extension is 0.10 m

(ii) Use the equation $\text{Energy stored} = \frac{1}{2} ke^2$ to calculate the energy stored at an extension of 0.10 m.

Q2 The graph below shows how the p.d. V across a capacitor varies with the charge stored by the capacitor.

[Fig 4C Charge v p.d. for a capacitor](#)

(a) Determine the capacitance C of the capacitor.

(b) (i) Use the graph to determine the energy stored in the capacitor when the p.d. is 10.0 V.

(ii) Use the equation $Energy\ stored = \frac{1}{2} CV^2$ to calculate the energy stored at a p.d. of 10.0 V.

Q3 The graph below shows how the gravitational field strength g varies with distance r from the centre of a planet.

[Fig 4D \$g\$ v \$r\$ a spherical planet](#)

(a) Show that the change of gravitational potential energy of a 1 kg mass when it is moved from the surface of the planet to infinity is 16 MJ.

(b) Hence show that the escape speed from the planet is 5600 m s^{-1} .

D. TRIGONOMETRY

D1. Sin , cos and tan (p117)

[Fig 4E](#)

Q1 In Fig 4E, calculate

(a) AB if (i) $AC = 5.0\text{ m}$ and $\theta = 30^\circ$, (ii) $BC = 8.0\text{ m}$ and $\theta = 60^\circ$

(b) BC if (i) $AC = 4.0\text{ m}$ and $\theta = 50^\circ$, (ii) $AB = 2.0\text{ m}$ and $\theta = 25^\circ$

(c) AC if (i) $AB = 3.0\text{ m}$ and $\theta = 70^\circ$, (ii) $BC = 2.5\text{ m}$ and $\theta = 45^\circ$

Q2 Use Fig 4E to explain why

(a) (i) $\sin \theta = \cos (90 - \theta)$, (ii) $\cos \theta = \sin (90 - \theta)$,

(b) (i) $\tan \theta = \frac{\sin \theta}{\cos \theta}$, (ii) $\tan 45 = 1$

Q3 Complete the Table below which refers to the right-angled triangle shown in Fig 4F.

[Fig 4F](#)

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
$\theta / ^\circ$	30.0			50			60
AB / m		8.0	4.0		12.0	3.0	
BC / m	6.0	4.0		8.0	8.0		
AC / m			5.0			9.0	4.0

[Click here for answers to D1](#)

D2. Sine wave formulae (p274, p286)

Q1 For a frequency of 100 Hz, calculate (i) $2\pi ft$ in radians , (ii) $\sin(2\pi ft)$ when

(a) $t = 1.0$ ms, (b) 2.0 ms , (c) 2.5 ms

Q2 An alternating voltage varies with time (in seconds) in accordance with the equation

$$V = V_0 \sin(2\pi ft)$$

(a) For $f = 50$ Hz and $V_0 = 12.0$ volts, calculate the voltage when time t is (i) 2.0 ms,

(ii) 4.0 ms , (iii) 7.5 ms , (iv) 16 ms (v) 25 ms ,

(b) For $f = 1000$ Hz and $V_0 = 5.0$ volts , calculate the least value of time t when V is equal to

(i) 2.5 V, (ii) 3.0 V , (iii) 4.0 V , (iv) 5.0 V

[Click here for answers to D2](#)

D3. Circular measures

Q1 Convert each of the following angles to radians ;-

(a) (i) 30° , (ii) 45° , (iii) 90° , (iv) 135° , (v) 180°

(b) Calculate the length s of an arc of a circle of radius 0.20 m that subtends an angle to the centre of the circle (a) (i) 30° , (ii) 45° , (iii) 90° , (iv) 135° , (v) 180°

Q2 Calculate the angle in (i) radians, (ii) degrees subtended to the centre of a circle of radius 0.50 m by an arc along the circle of length (a) 0.10 m , (b) 0.20 m, (c) 0.50 m , (d) 2.0 m

Q3 Calculate the radius of a circle which contains a segment with an arc of length 0.15 m that subtends an angle of (a) 10° , (b) 30° to the centre of the circle.

[Click here for answers to D3](#)

E. USING A CALCULATOR

E1. Standard form and powers of ten (p3)

Q1(a) Write down the symbol for each of the following prefixes and state its value as a power of ten ;-

(i) nano- (ii) micro- (iii) milli- (iv) kilo- (v) mega- (vi) giga-

(b) Write each of the following measurements in standard form in the stated unit;-

(i) 300,000 km /s in m s^{-1} , (ii) 31, 000, 000 s in s , (iii) 590 nm in metres

Q2 For each of the following pairs of numbers , calculate (i) their sum , (ii) their difference and express the answer in standard form ;-

(a) 3615 , 2132 (b) 1.65×10^4 , 12328 (c) 6.75×10^{19} , 5.41×10^{19}

(d) 3.49×10^6 , 4.5×10^5 (e) 3.46×10^{-6} , 2.15×10^{-6}

Q3 For each of the following pairs of numbers , calculate (i) their product (ii) the first number divided by the second number and express the answer in standard form to 3 significant figures ;-

(a) 3615 , 2132 (b) 1.65×10^4 , 12328 (c) 6.75×10^{19} , 5.41×10^{19}

(d) 3.49×10^6 , 4.5×10^5 (e) 3.46×10^{-6} , 2.15×10^{-6}

[Click here for answers to E1](#)

E2. Raising a number to a given power (p72, p381)

Q1 (a) Raise each of the following numbers to the fourth power and express the answer in standard form to 2 significant figures ;-

(i) 2.5 , (ii) 3.6×10^3 , (iii) 4.5×10^{10} , (iv) 7.4×10^{-3} , (v) 0.35

(b) Calculate the cube root of each of the numbers above and express the answer in standard form to 3 significant figures.

Q2 Raise (a) 4.6×10^8 (b) 6.7×10^{-10} to each of the following powers and express the answer in standard form to 3 significant figures.

(i) 0.2 (ii) 0.5 (iii) 2.5 (iv) $1/4$ (v) $2/5$

Q3 In the equation $p q^n = 100$, calculate and express the answer in standard form to 3 significant figures

(a) p when $q = 120$ and $n =$ (i) 4 (ii) 0.5 (iii) 1.4 (iv) -1.67 (v) -0.5

(b) q when $p = 40$ and $n =$ (i) 2 (ii) -0.5 (iii) 1.5 (iv) -3 (v) 10

[Click here for answers to E2](#)

E3. Using the 'ln' key and the 'e' key of a calculator (p215, p 336)

Q1 Calculate (a) e^x , (b) e^{-x} and express the answer in standard form to 3 significant figures for $x =$ (i) 0.5, (ii) 1 , (iii) 1.5 , (iv) 2.0 , (v) 2.5

Q2 Calculate $\ln x$ and express the answer in standard form to 3 significant figures for

(a) $x =$ (i) 0.01 , (ii) 0.1 , (iii) 1.0 , (iv) 10 , (v) 100

(b) $x =$ (i) 0.03 , (ii) 0.3 , (iii) 3.0 , (iv) 30 , (v) 300

Q3 (a) Calculate e^{px} and express the answer in standard form to 3 significant figures for $x = 1.5$ and $p =$ (i) -1.5, (ii) 0 , (iii) 1.5

(b) Calculate $\ln qx$ and express the answer in standard form to 3 significant figures for $x = 10$ and $q =$ (i) 0.04 , (ii) 0.40 , (iii) 4.0

[Click here for answers to E3](#)

F. VECTORS new

F1. Resolving a vector (p115-116)

Q1 (a) On a coordinate grid , draw to scale and measure the x and y components of each of the following vectors ;-

(i) a displacement vector of magnitude 30 m at an angle of 60° North of due East ,

(ii) a force vector of magnitude 100 N at an angle of 70° North of due West,

(b) Check your answers to (a) (i) and (ii) by calculating the x and y components for each of the above vectors.

Q2 (a) Fig 4G shows a force vector of magnitude F at angle θ to the x-axis .

[Fig 4G](#)

Calculate the x and y components of the vector if (i) $F = 10 \text{ N}$ and $\theta = 30^\circ$ (ii) $F = 8.0 \text{ N}$ and $\theta = 70^\circ$

(b) In the diagram above , calculate the magnitude F and the y-component if (i) $\theta = 60^\circ$ and the x-component = 8.0 N , (ii) $\theta = 45^\circ$ and the x-component = 6.0 N

Q3 An airport is located 10.0 km due North of the centre of a city. An aircraft takes off from the air port and passes over a prominent landmark which is 30.5 km in a direction 35° East of due North from the airport .

(a) (i) On a coordinate grid , using an appropriate scale, mark the locations of the airport , the city centre and the landmark.

(ii) Determine how far East and how far North the landmark is from the city centre.

(b) When the aircraft passes over the landmark, it is travelling on a level flight path at a constant velocity of 120 m s^{-1} in a direction 70° East of due North.

(i) Resolve the aircraft's velocity into a component due East and a component due North.

(ii) Determine how far East and how far North the aircraft is from the city centre 100 s after passing over the landmark.

[Click here for answers to F1](#)

F2. Vector addition (p115-116)

Q1 By means of a scale drawing , determine the resultant of

(a) a force F_1 of 6.0 N acting due East and a force F_2 of 10.0 N acting due North on a point object ,

(b) a force F_1 of 6.0 N acting due East and a force F_2 of 10.0 N acting 30° east of due North acting on a point object.

Q2 Check your answers to Q1 by means of calculations.

Q3 (a) By means of a scale drawing , determine the resultant of a force of 6.0 N and a force of 9.0 N acting at an angle of (i) 60° , (ii) 135° to one another,

(b) Check your answers to (a) by means of calculations.

[Click here for answers to F2](#)

F3 i and j vectors (p117)

Q1 (a) Represent each of the following vectors on the same coordinate grid :- (i) $2.0 \mathbf{i} + 3.0 \mathbf{j}$

(ii) $4.0 \mathbf{i} + 2.5 \mathbf{j}$

(b) Write down in i,j form (i) the sum, (ii) the difference $\mathbf{a(i)} - \mathbf{a(ii)}$ of the two vectors above

Q2 (a) Express each of the two forces in Q1(a) in i, j form and hence express the resultant of the two forms in i, j form. Hence determine the magnitude of the resultant .

(b) Express each of the two forces in Q1(b) in i, j form and hence express the resultant of the two forms in i, j form. Hence determine the magnitude of the resultant .

Q3 (a) Calculate the magnitude of each of the force vectors , given its i and j components in newtons :- (i) $3.0 \mathbf{i} + 4.5 \mathbf{j}$ (ii) $2.4 \mathbf{i} - 5.5 \mathbf{j}$

(b) Calculate the sum of the above two force vectors, expressing your answer (i) in i, j form , (ii) as a magnitude and an angle to the x-axis .

(c) Calculate the difference ,(i) - (ii) , of the above two force vectors, expressing your answer (i) in i, j form , (ii) as a magnitude and an angle to the x-axis .

[Click here for answers to F3](#)