

3. FURTHER PROOFS

1. Proof of the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ (Textbook p44)

[Fig 3A The lens formula](#)

In Fig 3A , the ratio $\frac{\text{image height}}{\text{object height}} = \frac{v}{u}$

The ratio is also equal to $\frac{v-f}{f}$ (similar triangles)

$$\therefore \frac{v}{u} = \frac{v-f}{f}$$

$$vf = vu - uf$$

$$\therefore vf + uf = vu$$

Dividing each term by uvf therefore gives the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

2. Proof that the moment of a horizontal rule of weight W and length L about one end = $\frac{1}{2}WL$ (Textbook p122)

[Fig 3B Centre of gravity of a rule](#)

Consider the rule divided along its length into n equal elements of length δx , where n is a large even number.

The weight of each element = W / n .

$$\text{The moment of the first element from the pivot} = \frac{W}{n} \times 0.5 \delta x = \frac{W \delta x}{2n}$$

$$\text{The moment of the second element from the pivot} = \frac{W}{n} (0.5 \delta x + \delta x) = \frac{3 W \delta x}{2n}$$

$$\text{The moment of the } r^{\text{th}} \text{ element from the pivot} = (2r - 1) \frac{W \delta x}{2n}$$

$$\therefore \text{The total moment about the pivot} = \{ 1 + 3 + \dots + (2n-1) \} \frac{W \delta x}{2n}$$

It can be shown that if n is an even number, $\{ 1 + 3 + \dots + (2n-1) \} = n^2$

$$\text{Hence the total moment about the pivot} = n^2 \frac{W\delta x}{2n} = W \frac{n \delta x}{2} = W \frac{L}{2}$$

For other regular shapes, the total moment is determined using integration which is a more generalised mathematical method of adding the contributions of each element.