Introduction to algebra

1 As part of a chemical manufacturing process a two-gas mixture increased in volume by 15.9%. If one of the gases increased in volume by 4% and the other increased in volume by 18% what were the proportions of the mixture to begin with?

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2 A company that develops earth moving equipment finds that the length of time taken for a test vehicle to complete a journey varies directly as the distance and inversely as the velocity. The velocity varies directly as the square root of the amount of fuel consumed per mile and inversely as the number of loading units imposed on the vehicle. In a journey of 25 miles in half an hour with 18 loading units imposed, 10 litres of fuel are required. How much fuel is consumed in a journey of 21 miles in 28 minutes with an imposition of 16 loading units?

Solutions Working

3 Bronze, being an alloy of copper, zinc and tin, contains 80% copper, 4% zinc and 16% tin. Molten brass, which is also an alloy, is mixed with molten bronze in the ratio of 3 parts brass to 5 parts bronze and the resultant mixture is found to consist of 74% copper, 16% zinc and 10% tin. What are the percentages of the constituents of brass?

Solutions Working

Solutions

1 17 : 3

Questions Working

2 5·376 litres

Questions Working

3 Copper 64%, Zinc 36%

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Working

1 Let V_1 and V_2 be the original volumes of each of the two gases. Then after the increase in volume of the mixture we see that:

$$V_1 \times 1{\cdot}18 + V_2 \times 1{\cdot}04 = (V_1 + V_2) \times 1{\cdot}159$$

Dividing through by V_2 yields:

$$\frac{V_1}{V_2} \times 1{\cdot}18 + 1{\cdot}04 = \left(\frac{V_1}{V_2} + 1\right) \times 1{\cdot}159 = \frac{V_1}{V_2} \times 1{\cdot}159 + 1{\cdot}159$$

That is:

$$\frac{V_1}{V_2} \times (1{\cdot}18 - 1{\cdot}159) = 1{\cdot}159 - 1{\cdot}04$$

and so

$$\frac{V_1}{V_2} = \frac{1{\cdot}159 - 1{\cdot}04}{1{\cdot}18 - 1{\cdot}159} = \frac{17}{3}$$

Therefore:

$$V_1 : V_2 = 17 : 3$$

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2 Let t denote the time in hours, d the distance in miles, v the velocity in miles per hour, f the quantity of fuel consumed per mile in litres and n is the number of loading units. Translating the question into symbolic form we see that:

The time taken varies directly as the distance and inversely as the velocity so:

$$t \propto \frac{d}{v}$$

The velocity varies directly as the square root of the amount of fuel consumed per mile and inversely as the number of loading units so:

$$v \propto \frac{\sqrt{f}}{n}$$

Therefore:

$$t \propto \frac{nd}{\sqrt{f}} \quad \text{and so} \quad t = k\frac{nd}{\sqrt{f}}$$

where k is the proportionality constant.

We are given that in a journey of 25 miles in half an hour with 18 loading units imposed, 10 litres of fuel are required so that:

$$\frac{1}{2} = k\frac{18 \times 25}{\sqrt{10/25}} \quad \text{therefore} \quad k = \frac{\sqrt{10}}{2 \times 5 \times 25 \times 18} \quad \text{giving} \quad t = \frac{\sqrt{10}nd}{4500\sqrt{f}}$$

To determine how much fuel is consumed in a journey of 21 miles in 28 minutes with an imposition of 16 loading units we substitute:

$n = 16$, $d = 21$ and $t = 28/60$ to obtain:

$$\sqrt{f} = \frac{\sqrt{10}nd}{4500t} = \frac{\sqrt{10} \times 16 \times 21 \times 60}{4500 \times 28} = 0{\cdot}50596\ldots$$

giving $f = 0{\cdot}256$ litres per mile

The quantity of fuel consumed in 21 miles is therefore:

$21 \times 0{\cdot}256 = 5{\cdot}376$ litres

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3 In 100 g of brass there is:

x g of copper
y g of zinc
z g of tin

So if 300 g of brass is mixed with 500 g of bronze to create 800 g of the mixture then the resultant mixture contains:

$3x + 5 \times 80 = 8 \times 74$ g of copper therefore $3x + 400 = 592$

and so $x = \dfrac{592 - 400}{3} = 64$

$3y + 5 \times 4 = 8 \times 16$ g of zinc therefore $3y + 20 = 129$

and so $y = \dfrac{128 - 20}{3} = 36$

$3z + 5 \times 16 = 8 \times 10$ g of tin therefore and so $z = 0$

The percentages of the constituents of brass are, therefore, 64% copper and 36% zinc.

Questions Solutions
