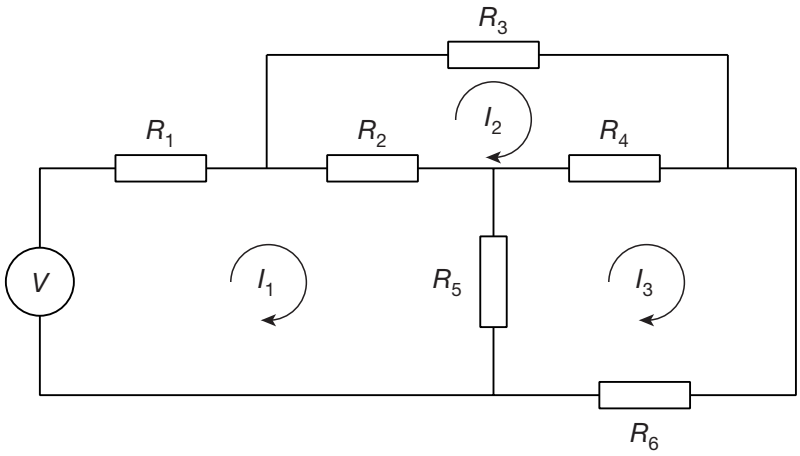
Linear equations and simultaneous linear equations

1 Finecut Tailors Ltd manufacture one style of overcoat and one style of jacket. Each overcoat requires 6 metres of cloth and 5 metres of lining and each jacket requires 4 metres of cloth and 2 metres of lining. Each week Finecut uses 600 metres of cloth and 360 metres of lining material in the manufacture of their overcoats and jackets. How many jackets and overcoats do Finecut manufacture in a week?

Solutions Working

2 Six resistors are connected as in the circuit shown with an applied DC of V volts.



Evaluate the loop currents shown when:

(a) Each resistor is equal to $100\,\Omega$ and $V = 100$ volts
(b) Each resistor is equal to $100\,\Omega$, except $R_5 = 0\,\Omega$ and $V = 100$ volts

Solutions Working

Solutions

1 Finecut manufacture 30 overcoats and 105 jackets each week.

Questions Working

2 (a) $I_1 = 0.50$ A, $I_2 = 0.25$ A and $I_3 = 0.25$ A
(b) $I_1 = 0.625$ A, $I_2 = 0.25$ A and $I_3 = 0.125$ A

Questions Working

Working

1 Let: A represent the number of overcoats made
B represent the number of jackets made

An overcoat requires 6 metres of cloth and a jacket requires 4 metres of cloth, therefore:

$$6A + 4B = 600$$

An overcoat requires 5 metres of lining and a jacket requires 2 metres of lining, therefore:

$$5A + 2B = 360$$

Solving simultaneously:

$$\begin{array}{lll} 6A + 4B = 600 & & 6A + 4B = 600 \\ 5A + 2B = 360 & \text{that is} & 10A + 4B = 720 \\ & \text{so that} & 4A \quad = 120 \quad \text{so} \quad A = 30 \end{array}$$

Then:

$$6A + 4B = 600 \quad \text{so} \quad 180 + 4B = 600 \quad \text{giving} \quad 4B = 600 - 180$$

therefore $B = 105$

Finecut manufacture 30 overcoats and 105 jackets each week.

Questions Solutions

2 Using Kirchoff's laws we have that:

$$R_1I_1 + R_2(I_1 - I_2) + R_5(I_1 - I_3) = V$$
$$R_3I_2 + R_2(I_2 - I_1) + R_4(I_2 - I_3) = 0$$
$$R_4(I_3 - I_2) + R_6I_3 + R_5(I_3 - I_1) = 0$$

That is:

$$(R_1 + R_2 + R_5)I_1 - R_2I_2 - R_5I_3 = V$$
$$-R_2I_1 + (R_2 + R_3 + R_4)I_2 - R_4I_3 = 0$$
$$-R_5I_1 - R_4I_2 + (R_4 + R_5 + R_6)I_3 = 0$$

(a) When $R_1 = 100\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 100\,\Omega$, $R_4 = 100\,\Omega$, $R_5 = 100\,\Omega$, $R_6 = 100\,\Omega$ and $V = 100$ volts then:

$$300I_1 - 100I_2 - 100I_3 = 100 \quad [1]$$
$$-100I_1 + 300I_2 - 100I_3 = 0 \quad [2]$$
$$-100I_1 - 100I_2 + 300I_3 = 0 \quad [3]$$

So that:

$$300I_1 - 100I_2 - 100I_3 = 100 \quad [1]$$
$$-300I_1 + 900I_2 - 300I_3 = 0 \quad 3 \times [2]$$

and adding gives $800I_2 - 400I_3 = 100$ [4]

and

$$300I_1 - 100I_2 - 100I_3 = 100 \quad [1]$$
$$-300I_1 - 300I_2 + 900I_3 = 0 \quad 3 \times [3]$$

and adding gives $-400I_2 + 800I_3 = 100$ [5]

Then:

$$800I_2 - 400I_3 = 100 \quad [4]$$
$$-800I_2 + 1600I_3 = 200 \quad 2 \times [5]$$

and adding gives $1200I_3 = 300$

and so $I_3 = 0{\cdot}25$

Substituting in [4] gives:

$$800I_2 - 400(0{\cdot}25) = 800I_2 - 100 = 100$$

so that $800I_2 = 200$ giving $I_2 = 0{\cdot}25$ A

Substituting in [1] gives:

$$300I_1 - 100(0{\cdot}25) - 100(0{\cdot}25) = 100$$

so that $300I_1 = 150$ and so $I_1 = 0{\cdot}50$ A

(b) When $R_1 = 100\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 100\,\Omega$, $R_4 = 100\,\Omega$, $R_5 = 0\,\Omega$, $R_6 = 100\,\Omega$ and $V = 100$ volts then:

$$\begin{aligned} 200I_1 - 100I_2 &= 100 && [1] \\ -100I_1 + 300I_2 - 100I_3 &= 0 && [2] \\ -100I_2 + 200I_3 &= 0 && [3] \end{aligned}$$

So that:

$$\begin{aligned} 200I_1 - 100I_2 &= 100 && [1] \\ -200I_1 + 600I_2 - 200I_3 &= 0 && 2 \times [2] \end{aligned}$$

and adding gives $500I_2 - 200I_3 = 100$ [4]

and

$$\begin{aligned} 500I_2 - 200I_3 &= 100 && [4] \\ -500I_2 + 1000I_3 &= 0 && 5 \times [3] \end{aligned}$$

and adding gives $800I_3 = 100$

and so $I_3 = 0{\cdot}125$ A

Substituting in [4] gives:

$500I_2 - 200(0{\cdot}125) = 500I_2 - 25 = 100$

so that $500I_2 = 125$ giving $I_2 = 0{\cdot}25$ A

Substituting in [1] gives:

$200I_1 - 100(0{\cdot}25) = 2I_1 - 25 = 100$

so that $200I_1 = 125$ and so $I_1 = 0{\cdot}625$ A

Questions Solutions