Polynomial equations

1 A chemical reaction is dependent upon the presence of a catalyst. If there is no catalyst present there will be no reaction but if there is too much catalyst present the reaction breaks down and the product will be so contaminated that it is of no use. A chemist finds that in a given production process the reaction rate, the amount of product produced per unit of catalyst, is related to the amount x of catalyst present according to the equation $r(x) = 160 - x$. If

$$W(x) = \frac{x^2}{3} + 50x + 1400$$

is the amount of impure or waste product produced, $R(x)$ is the total amount of pure and impure product produced and $P(x)$ is the amount of pure product produced:

(a) Find an equation for $R(x)$ and $P(x)$, both in terms of x.
(b) Find the levels of catalyst to 1 dp at which the process breaks down and no pure product is produced, and the associated reaction rates.
(c) Determine the catalyst levels to 1 dp when the amount of pure product produced is 200.
(d) The chemist now adopts a new reaction process from which the total amount of impure product produced is a linear function of the amount of catalyst x. If, with this process, the reaction breaks down at 10 and 70 units of catalyst, determine the new amount of pure product as a function of x.

Solutions Working

2 A manufacturer has a steady demand for an engineering product of 600 units per annum and these are produced on an assembly line in a series of equal sized batches. Production is continuous and linear and as soon as one batch is complete another batch is started. It takes 6 man-hours to produce each unit and it takes 10 man-hours to prepare for each batch of units to be produced, regardless of the size of the batch. From past experience the company allows for delay to occur duriing preparation. The total time per year allowed in preparation is calculated at 20% of the time taken to produce a single unit multiplied by half the batch size. If:

The batch size is denoted by b units
The preparation delay for one unit per year is denoted by h man-hours
The time taken to prepare for the production of each batch is denoted by s man-hours
The annual demand for the product is denoted by d units
The time taken to produce each item is t man-hours
The total annual time expended in producing the units in man-hours is T where:

$$T = \text{annual production time} + \text{annual preparation time} + \text{annual preparation delay time}$$

(a) Show that:

$$T = td + \frac{sd}{b} + \frac{hb}{2}$$

(b) Find the total time T if the batch size is

(i) 25 items
(ii) 50 items

Solutions Working

3 In Problem 2 find the batch size when the total time $T = 3720$ hours.

Solutions Working

Solutions

1

(a) $R(x) = 160x - x^2$, $P(x) = -\dfrac{4x^3}{3} + 110x - 1400$

(b) $x = 66{\cdot}8,\ 15{\cdot}7,\ r(66{\cdot}8) = 93{\cdot}2$ and $r(15{\cdot}7) = 144{\cdot}3$

(c) $x = 63{\cdot}7,\ 18{\cdot}9$

(d) $P(x) = -x^2 + 80x - 700$

Questions Working

2 (b) (i) $T = 3855$

(ii) $T = 3750$

Questions Working

3 100

Questions Working

Working

1 (a) The amount of product produced is the amount of catalyst present multiplied by the amount of product produced per unit of catalyst. That is

$$R(x) = xr(x) = x(160 - x) = 160x - x^2$$

The amount of pure product is then the difference between the total amount of product and the amount of waste product. That is:

$$\begin{aligned} P(x) &= R(x) - W(x) \\ &= (160x - x^2) - \left(\frac{x^2}{3} + 50x + 1400\right) \\ &= -\frac{4x^2}{3} + 110x - 1400 \end{aligned}$$

(b) The reaction breaks down when the amount of pure product is zero, that is when:

$$-\frac{4x^2}{3} + 110x - 1400 = 0$$

This happens when

$$x = \frac{-110 \pm \sqrt{(110)^2 - 4 \times (-4/3) \times (-1400)}}{2 \times (-4/3)}$$

That is when:

$$x = \frac{-110 \pm 68.07}{-(8/3)} = 66{\cdot}78,\ 15{\cdot}72,$$

that is 66·8 and 15·7 units of catalyst.

This translates into the reaction rates of:

$$r(66{\cdot}8) = 160 - 66{\cdot}8 = 93{\cdot}2$$

and $r(15{\cdot}7) = 160 - 15{\cdot}7 = 144{\cdot}3$

(c) When $P(x) = -\dfrac{4x^2}{3} + 110x - 1400 = 200$ then:

$$-\frac{4x^2}{3} + 110x - 1600 = 0$$

and so

$$\begin{aligned} x &= \frac{-110 \pm \sqrt{110^2 - 4 \times (-4/3) \times (-1600)}}{-(8/3)} \\ &= \frac{-110 \pm 59{\cdot}72}{-(8/3)} = 63{\cdot}65,\ 18{\cdot}86 \end{aligned}$$

that is $x = 63{\cdot}7$ and $x = 18{\cdot}9$.

(d) The chemist now adopts a new reaction process from which the total amount of impure product produced is a linear function of the amount of catalyst x. If, with this process, the reaction breaks down at 10 and 70 units of catalyst, determine the new amount of pure product as a function of x.

$$\begin{aligned} P(x) &= R(x) - W(x) \\ &= (160x - x^2) - (ax + b) \\ &= -x^2 + (160 - a)x - b \\ &= -(x - 10)(x - 70) \end{aligned}$$

so

$$-x^2 + (160 - a)x - b = -(x - 10)(x - 70) = -x^2 + 80x - 700$$

therefore

$(160 - a)x - b = 80x - 700$
giving $\quad a = 80,\ b = 700$
and hence $\quad W(x) = 80x + 700$

and so:

$$\begin{aligned} P(x) &= R(x) - W(x) \\ &= (160x - x^2) - (80x + 700) \\ &= -x^2 + 80x - 700 \end{aligned}$$

Questions Solutions

2 (a) The annual production time is the production time per item multiplied by the number of items produced. That is td man-hours.

The annual preparation time is the time to prepare for one batch s multiplied by the number of batches produced in a year. The number of batches produced in a year is obtained from the number of units made per year divided by the number in a batch, that is d/b. So that the annual batch preparation time is:

$$\frac{sd}{b} \text{ man-hours}$$

The total preparation delay for one year is given by the preparation delay for one unit, which is h, multiplied by half the batch size $b/2$. The annual production delay is then:

$$\frac{hb}{2} \text{ man-hours}$$

This gives the total annual man-hours time as:

$$T = td + \frac{sd}{b} + \frac{hb}{2}$$

(b) (i) The batch size is $b = 25$

The preparation delay for one unit is

$$h = 6 \times \frac{20}{100} = 1{\cdot}2 \text{ man-hours}$$

The batch set-up time is $s = 10$ man-hours

The annual demand for the product is $d = 600$

The time taken to produce one unit of product is $t = 6$ man-hours therefore:

$$\begin{aligned} T &= 6 \times 600 + \frac{10 \times 600}{25} + \frac{1{\cdot}2 \times 25}{2} \text{ man-hours} \\ &= 3855 \text{ man-hours} \end{aligned}$$

(ii) The batch size is $b = 50$
The preparation delay for one unit is

$$h = 6 \times \frac{20}{100} = 1{\cdot}2 \text{ man-hours}$$

The batch set-up time is $s = 10$ man-hours
The annual demand for the product is $d = 600$
The time taken to produce one unit of product is $t = 6$ man-hours therefore:

$$T = 6 \times 600 + \frac{10 \times 600}{50} + \frac{1{\cdot}2 \times 50}{2} \text{ man-hours}$$
$$= 3750 \text{ man-hours}$$

Questions Solutions

3

When $T = 3720$ then, since $T = td + \frac{sd}{b} + \frac{hb}{2}$, we have:

$$3720 = 6 \times 600 + \frac{10 \times 600}{b} + \frac{1{\cdot}2 \times b}{2}$$

That is:

$$3720b = 3600b + 6000 + 0{\cdot}6b^2 \quad \text{so that} \quad 0{\cdot}6b^2 - 120b + 6000 = 0$$

Therefore:

$$b^2 - 200b + 10\,000 \quad \text{giving} \quad b = \frac{200 \pm \sqrt{40\,000 - 40\,000}}{2} = 100 \text{ units}$$

Questions Solutions
