

# Trigonometry

1 Two waveforms:

$$f(t) = 4 \cos \omega t \text{ and } g(t) = 2 \sin \omega t$$

are detected by an oscilloscope and their sum is presented on the screen. Find the equation of the resulting shape.

Solutions

Working

2 The dimensions of a building extension with a sloping roof are determined by the standard sizes of timber available. Roof beams are standardized at 3 metres in length and building regulations state that the roof must be at 35 degrees to the horizontal.

- What is the maximum possible width of the extension?
- How much higher is the upper end of the roof beam above the lower end?

Solutions

Working

3 (a) How many 3 cm × 4 cm × 5 cm triangular blanks can be cut out of a sheet of steel measuring 2000 mm by 1000 mm assuming no steel is lost by a cut?

(b) What is your answer if 1.25 mm of steel is lost in every cut?

Solutions

Working

4 Use your spreadsheet to draw the graphs of each of the following:

- $f(t) = 5e^{-0.3t} \cos 2t$  for  $-5 \leq t \leq 15$
- $f(t) = 5e^{0.3t} \cos 2t$  for  $-5 \leq t \leq 15$
- $f(t) = \cos t + \sin t$  for  $-5 \leq t \leq 15$  in steps of 0.2
- $f(t) = \cos t + \sin \pi t$  for  $-5 \leq t \leq 15$  in steps of 0.2

Compare graph (a) with graph (b) and describe what is happening in each case.

Compare graph (c) with graph (d) and describe what is happening in each case.

Solutions

Working

## Solutions

**1**  $f(t) + g(t) = 29 \cos(\omega t - \alpha)$

Questions

Working

- 2** (a) 4.9 metres  
(b) 1.72 metres

Questions

Working

- 3** (a) 1650  
(b) 1536

Questions

Working

- 4** The plot of graph (a) displays a periodic function with decreasing amplitude whereas the plot of graph (b) displays a periodic function with increasing amplitude.

The plot of graph (c) displays a regular periodic function whereas the plot of graph (c) displays a non-periodic function. When two periodic functions are added together they will form into a third periodic function only if their frequencies are rational. The frequency of is which is irrational.

Questions

Working

## Working

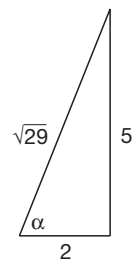
**1**  $f(t) + g(t) = 5 \cos \omega t + 2 \sin \omega t$

$$= \sqrt{29} \left( \frac{5}{\sqrt{29}} \cos \omega t + \frac{2}{\sqrt{29}} \sin \omega t \right)$$

$$\text{where } \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$= \sqrt{29} (\cos \alpha \cos \omega t + \sin \alpha \sin \omega t)$$

$$= \sqrt{29} \cos(\omega t - \alpha)$$



Questions

Solutions

- 2** (a) For a roof beam of length 3 metres inclined at an angle of 35 degrees to the horizontal the horizontal extent of the beam is given as:

$$3 \cos 35^\circ = 2.457 \text{ metres to 3 dp}$$

so the best estimate of the horizontal extent is 2.45 metres.

The maximum width, therefore, of the extension is then

$$2 \times 2.45 = 4.9 \text{ metres.}$$

- (b) For a roof beam of length 3 metres inclined at an angle of 35 degrees to the horizontal the vertical extent of the beam is given as:

$$3 \sin 35^\circ = 1.721 \text{ metres to 3 dp}$$

so the best estimate of the vertical extent is 1.72 metres.

Questions

Solutions

**3**

- (a) A triangle 3 cm × 4 cm × 5 cm is a right-angled triangle where the longer side is the hypotenuse.

cutting the 100 cm side into units of 4 will yield 25 pieces with nothing left over

cutting the 200 cm side into units of 3 will yield 66 pieces with  $2 \text{ cm} \times 100 \text{ cm} = 200 \text{ cm}^2$  left over

cutting the 200 cm side into units of 4 will yield 50 pieces with nothing left over

cutting the 100 cm side into units of 3 will yield 33 pieces with  $1 \text{ cm} \times 200 \text{ cm} = 200 \text{ cm}^2$  left over

Either way the total number of triangles is:

$$50 \times 33 = 1650 \text{ or } 25 \times 66 = 1650$$

- (b) If cutting uses 1.25 mm = 0.125 cm) then, noting that there is one more piece than the number of cuts made:

cutting the 100 cm side into units of 4.125 will yield 24 pieces with 1.125 cm left over

cutting the 200 cm side into units of 3.125 will yield 64 pieces with 0.125 cm left over

cutting the 200 cm side into units of 4.125 will yield 48 pieces with 2.125 cm left over

cutting the 100 cm side into units of 3.125 will yield 32 pieces with 0.125 cm left over

Either way the total number of triangles will be:

$$24 \times 64 = 1536 \text{ or } 48 \times 32 = 1536$$

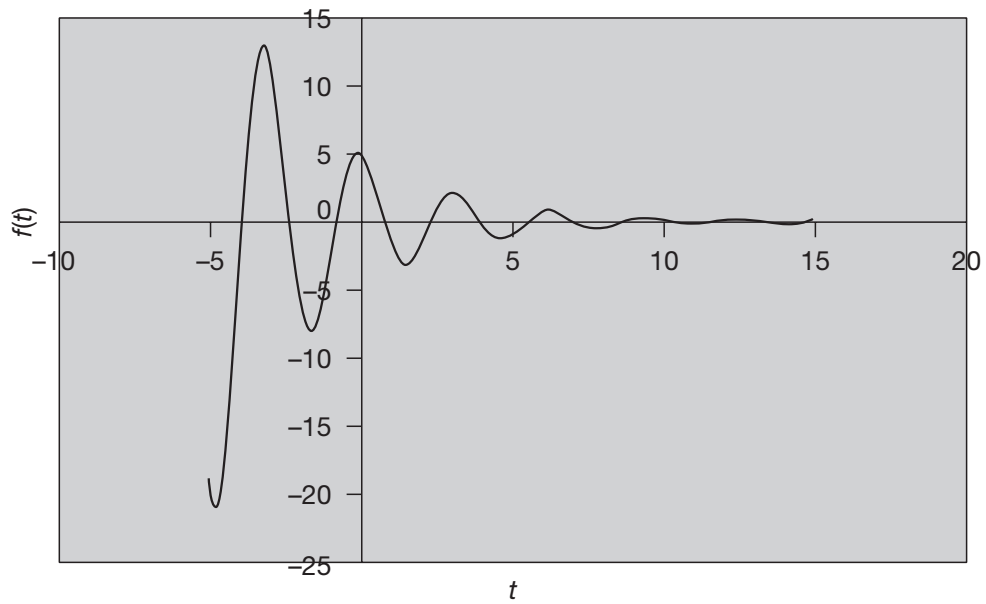
Questions

Solutions

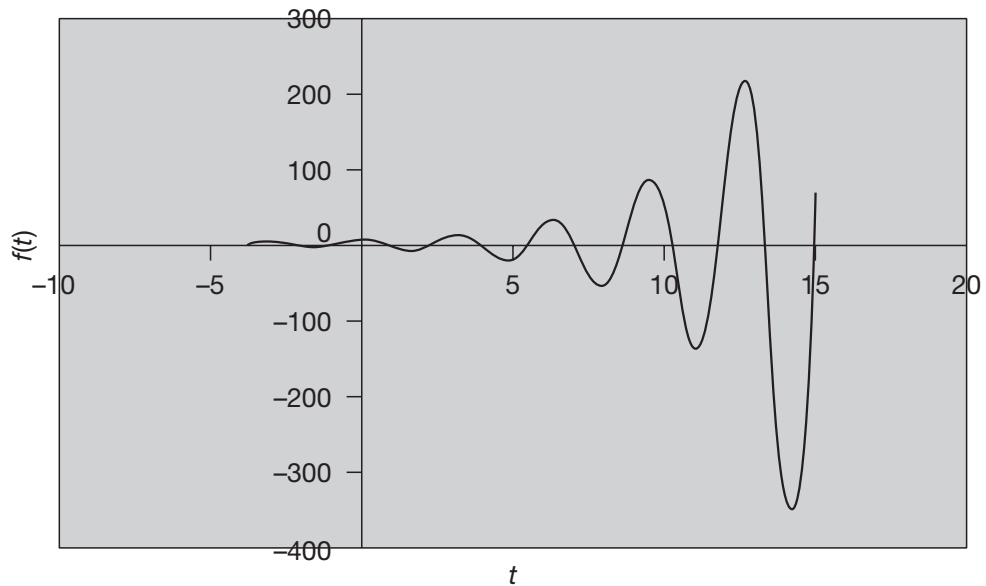
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**4**

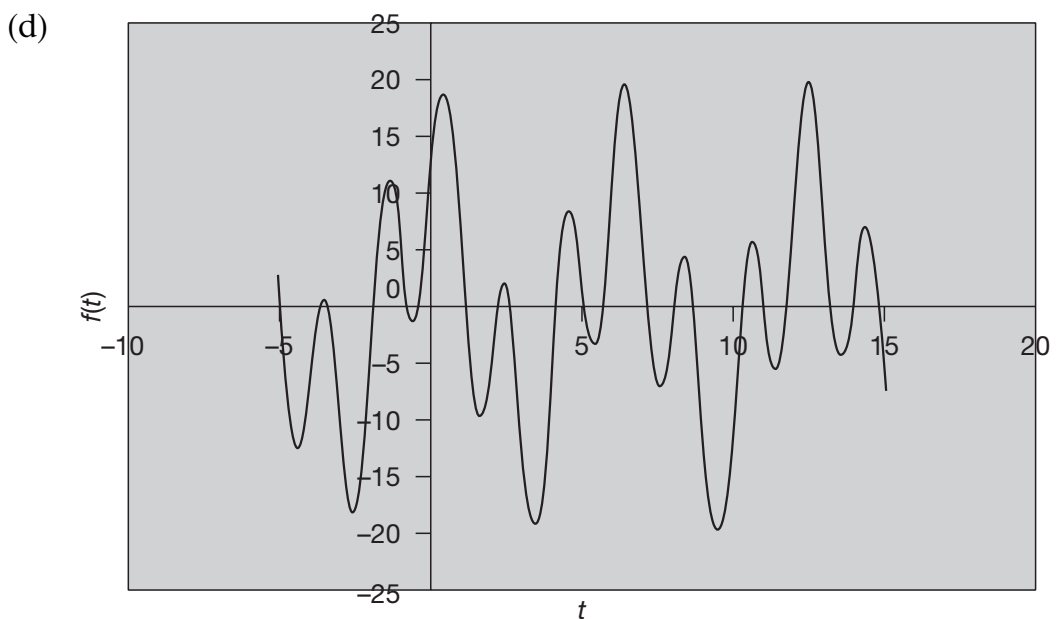
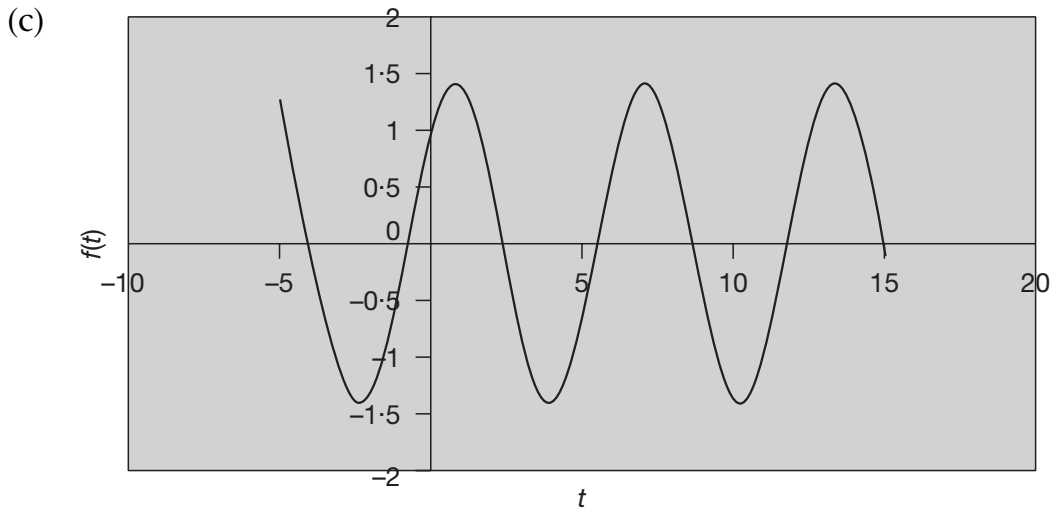
(a)



(b)



The plot of graph (a) displays a periodic function with decreasing amplitude whereas the plot of graph (b) displays a periodic function with increasing amplitude.



The plot of graph (c) displays a regular periodic function whereas the plot of graph (d) displays a non-periodic function. When two periodic functions are added together they will form into a third periodic function only if their frequencies are rational. The frequency of is which is irrational.

[Questions](#)

[Solutions](#)