Complex numbers 1

1 If ω is a complex cube root of unity find the values of:

(a) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

(b) $\begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega^2 \\ \omega^2 & \omega^3 & 1 \end{vmatrix}$

Solution

1 (a) 0
(b) 0

Working

1 (a) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 1\begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega\begin{vmatrix} \omega & 1 \\ \omega^2 & \omega \end{vmatrix} + \omega^2\begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$

$$= \left(\omega^3 - 1\right) - \omega(\omega^2 - \omega^2) + \omega^2\left(\omega - \omega^4\right)$$
$$= \omega^3 - 1 + \omega^3 - \omega^6$$
$$= 1 - 1 + 1 - 1^2 = 0 \quad \text{since } \omega^3 = 1$$

(b) $\begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega^2 \\ \omega^2 & \omega^3 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & \omega^2 \\ \omega^3 & 1 \end{vmatrix} - \omega^3\begin{vmatrix} \omega^3 & \omega^2 \\ \omega^2 & 1 \end{vmatrix} + \omega^2\begin{vmatrix} \omega^3 & 1 \\ \omega^2 & \omega^3 \end{vmatrix}$

$$\begin{aligned}
&= \left(1-\omega^5\right) - \omega^3(\omega^3-\omega^4) + \omega^2\left(\omega^6-\omega^2\right) \\
&= \left(1-\omega^2\right) - (1-\omega) + \omega^2\left(1-\omega^2\right) \quad \text{since } \omega^3 = 1 \\
&= 1-\omega^2-1+\omega+\omega^2-\omega^4 \\
&= 1-\omega^2-1+\omega+\omega^2-\omega \quad \text{since } \omega^3 = 1 \\
&= 0
\end{aligned}$$
