Determinants

1 In a certain DC network it was found that the linear equations satisfied by three independent branch currents i_1, i_2 and i_3 were as follows:

$$-2i_1 + 8i_2 - 3i_3 = 3$$
$$-i_1 - 3i_2 + 5i_3 = 5$$
$$4i_1 - 2i_2 - i_3 = 2$$

Use determinants to show that $i_3 = 2{\cdot}5$ A and then calculate i_1 and i_2.

Solution

1 $i_1 = 2{\cdot}0$ A to 1 dp
$i_2 = 1{\cdot}8$ A to 1 dp

Working

1 The matrix representation of these three equations that describe the DC network is:

$$\begin{pmatrix} -2 & 8 & -3 \\ -1 & -3 & 5 \\ 4 & -2 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$$

Using Cramer's rule to solve for i_3 we find that:

$$i_3 = \frac{\begin{vmatrix} -2 & 8 & 3 \\ -1 & -3 & 5 \\ 4 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} -2 & 8 & -3 \\ -1 & -3 & 5 \\ 4 & -2 & -1 \end{vmatrix}}$$

obtained by forming the determinant of the coefficient matrix which is the denominator; the numerator is the determinant of the coefficient matrix with the elements of the third column replaced by the elements of the right-hand side column matrix.

$$= \frac{-2\begin{vmatrix} -3 & 5 \\ -2 & 2 \end{vmatrix} - 8\begin{vmatrix} -1 & 5 \\ 4 & 2 \end{vmatrix} + 3\begin{vmatrix} -1 & -3 \\ 4 & -2 \end{vmatrix}}{-2\begin{vmatrix} -3 & 5 \\ -2 & -1 \end{vmatrix} - 8\begin{vmatrix} -1 & 5 \\ 4 & -1 \end{vmatrix} - 3\begin{vmatrix} -1 & -3 \\ 4 & -2 \end{vmatrix}}$$

$$= \frac{-2(-6+10) - 8(-2-20) + 3(2+12)}{-2(3+10) - 8(1-20) - 3(2+12)}$$

$$= \frac{-8+176+42}{-26+152-42}$$

$$= 2{\cdot}5$$

Substitution of this value into the first two equations of the three yields:

$$-2i_1 + 8i_2 - 7{\cdot}5 = 3$$
$$-i_1 - 3i_2 + 12{\cdot}5 = 5$$

That is:

$$-2i_1 + 8i_2 = 10{\cdot}5$$
$$-i_1 - 3i_2 = -7{\cdot}5$$

Multiplying the second equation by -2 gives:

$$-2i_1 + 8i_2 = 10{\cdot}5$$
$$2i_1 + 6i_2 = 15$$

Adding together we find:

$14i_2 = 25{\cdot}5$ so $i_2 = 25{\cdot}5/14 = 1{\cdot}8$ A to 1 dp

Substitution of this value into the first of the two equations above yields:

$-2i_1 + 8(1{\cdot}8) = 10{\cdot}5$ giving $i_1 = 2{\cdot}0$ A to 1 dp

Question Solution