Matrices

1 In the analysis of an electrical system the following matrix was derived:

$$\mathbf{A} = \begin{pmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

Verify that $\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ is an eigenvector with eigenvalue 2.

Working

1 The eigenvalues of matrix $\mathbf{A}$ are found by solving the equation $|\mathbf{A} - \lambda\mathbf{I}| = 0$. Now,

$$\mathbf{A} - \lambda\mathbf{I} = \begin{pmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and so

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \quad \text{becomes} \quad \begin{vmatrix} -2-\lambda & -8 & -12 \\ 1 & 4-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

That is, expanding about the first column:

$$(-2-\lambda)\begin{vmatrix} 4-\lambda & 4 \\ 0 & 1-\lambda \end{vmatrix} - \begin{vmatrix} -8 & -12 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

that is:

$$(-2-\lambda)(4-\lambda)(1-\lambda) + 8(1-\lambda) = 0$$

so that

$$\lambda(\lambda-2)(1-\lambda) = 0$$

so

$$\lambda = 0,\ 1,\ 2.$$

The eigenvector corresponding to the eigenvalue $\lambda = 2$ is then $\mathbf{x}$ where

$$\mathbf{Ax} = \begin{pmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2\mathbf{x} \quad \text{that is} \quad \begin{aligned} -2x_1 - 8x_2 - 12x_3 &= 2x_1 \\ x_1 + 4x_2 + 4x_3 &= 2x_2 \\ x_3 &= 2x_3 \end{aligned}$$

From this we can see that $x_3 = 0$ and that $x_1 = -2x_2$ so that, by choosing $x_2 = 1$,

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \quad \text{is an eigenvector with eigenvalue 2}$$

Question