Partial differentiation

1 The extension e of a uniformly tapering rod of length L when subjected to an axial load P is given by:

$$e = \frac{4PL}{\pi DdE}$$

where D and d are the diameters at each end and E is the modulus of elasticity. Write down a formula for the approximate error δe in terms of errors δD, δd and δP in the measured values of D, d and P respectively and the partial derivatives of e with respect to these variables.

If the maximum percentage errors in measuring D, d and P are $\pm 2\%$, $\pm 3\%$ and $\pm 4{\cdot}5\%$ respectively, use your formula to find the magnitude of the maximum possible percentage error in the calculated value of e.

Solution Working

Solution

1 9.5%

Question Working

Working

1

Given that $e = \dfrac{4PL}{\pi DdE}$ then the approximate error δe in terms of errors δD, δd and δP is given by the equation:

$$\begin{aligned}\delta e &\cong \frac{\partial e}{\partial D}\delta D + \frac{\partial e}{\partial d}\delta d + \frac{\partial e}{\partial P}\delta P\\ &= -\frac{4PL}{\pi D^2 dE}\delta D - \frac{4PL}{\pi Dd^2E}\delta d + \frac{4L}{\pi DdE}\delta P\\ &= -\frac{e}{D}\delta D - \frac{e}{d}\delta d + \frac{e}{P}\delta P\end{aligned}$$

Therefore:

$$\begin{aligned}\frac{\delta e}{e}\times 100 &= -\frac{\delta D}{D}\times 100 - \frac{\delta d}{d}\times 100 + \frac{\delta P}{P}\times 100\\ &= \mp 2 \mp 3 \pm 4{\cdot}5\end{aligned}$$

This means that the magnitude of the maximum possible percentage error in e is 9·5%.

Question Solution