

Curves and curve fitting

- 1 The amount of solute that will dissolve in a given volume of water depends on the temperature of the water. For a particular substance it is believed that the amount that will dissolve in 10 litres of water is governed by the relationship:

$$m = Ae^{k\theta}$$

where θ is the water temperature and A and k are constants. An experiment was performed to test this hypothesis and the following data was recorded:

Temperature	10	20	30	40	50	60	70
Mass	3.72	4.32	5.02	5.83	6.77	7.87	9.14

- (a) Plot a linear graph which confirms that the relationship is exponential as suspected
(b) Find the values of A and k .

Solution

Working

Solution

- 1 (b) $k = 0.015$, $A \cong 3.2$ to 1 dp

Question

Working

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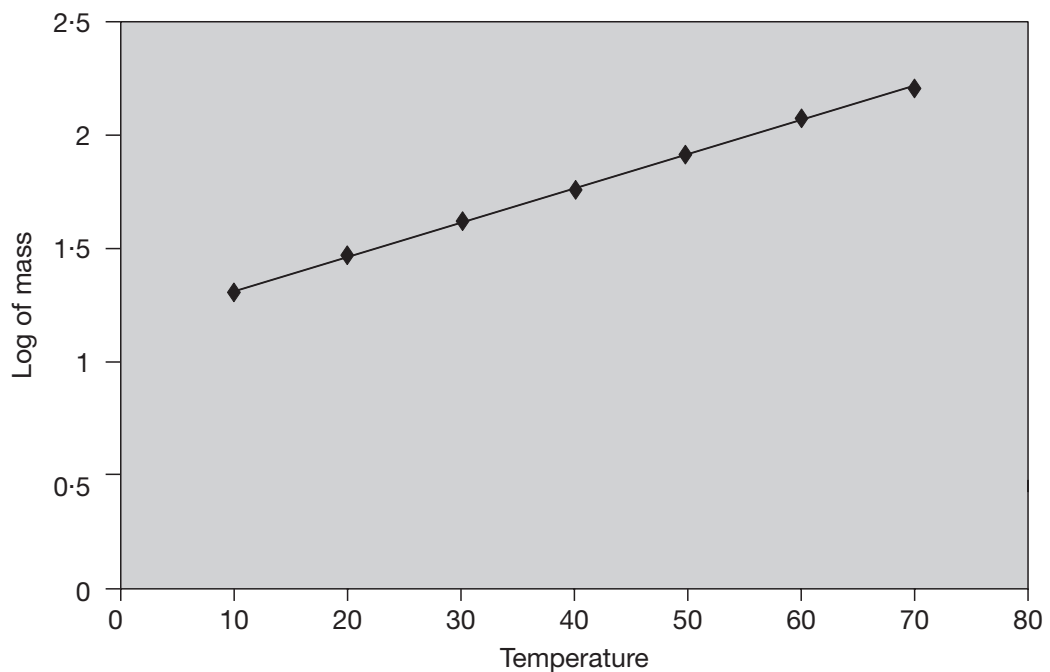
- 1 (a) To convert $m = Ae^{k\theta}$ into a linear form we take the natural logarithm of each side to obtain:

$$\ln m = \ln A + k\theta$$

So if we plot $\ln m$ against θ we should obtain a straight line with gradient k and vertical intercept $\ln A$. The table is therefore extended to read as:

Temperature	10	20	30	40	50	60	70
Mass	3.72	4.32	5.02	5.83	6.77	7.87	9.14
$\ln m$	1.31	1.46	1.61	1.76	1.91	2.06	2.21

giving the following graph which is clearly a straight line.

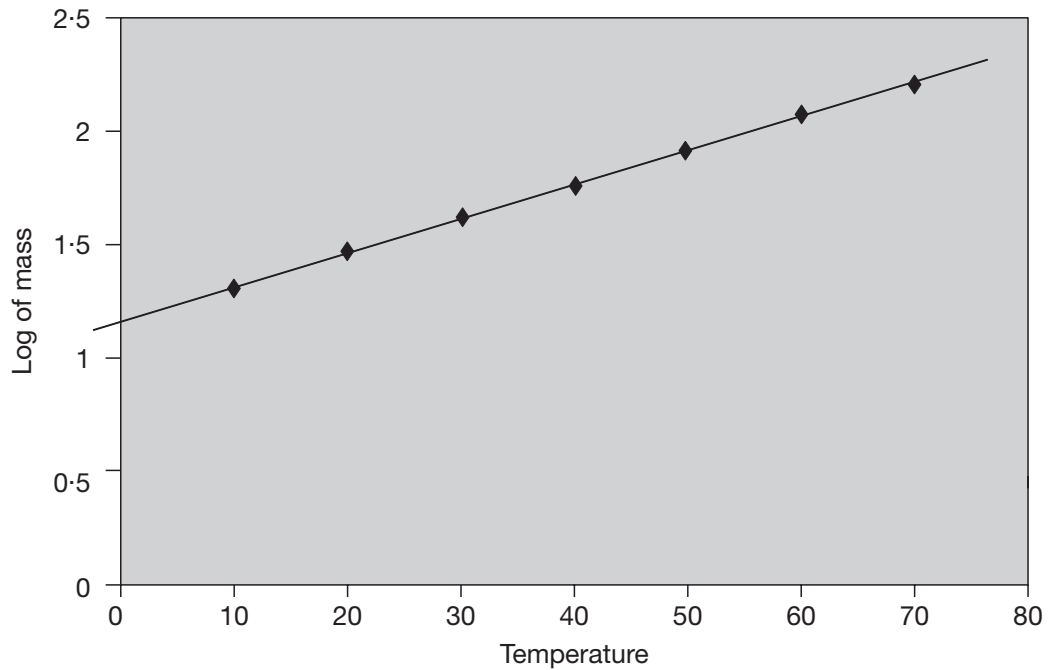


(b) The gradient is evaluated using the two end points:

$$k = \frac{2.21 - 1.31}{70 - 10} = 0.015$$

The vertical intercept requires the graph to be extended (as below) from which we deduce that the vertical intercept is at 1.15. That is:

$$\ln A = 1.15 \quad \text{and so} \quad A = e^{1.15} \cong 3.2 \text{ to 1 dp.}$$



Question

Solution